Is an absolute rest frame U, necessary for measurements involving translational/inertial motion within Special Relativity?

The expression for gamma (γ), the scaling factor for coordinate transformations is

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \tag{1}$$

with v the speed of the observer relative to U.

In the following the speed of the observer relative to U will be noted as a lower case letter corresponding to the upper case letter identifying the observer.

The coordinate transformations for A are

$$x_A = \gamma(x - vt) \tag{2}$$

and

$$t_A = \gamma(t - vx) \tag{3}$$

which can be rewritten with a = v/c and c=1, as

$$= x(1-a)/\sqrt{1-a^2} = x\sqrt{\frac{(1-a)}{(1+a)}} \tag{4}$$

and

$$= t(1-a)/\sqrt{1-a^2} = t\sqrt{\frac{(1-a)}{(1+a)}} \tag{5}$$

For a second observer B

$$x_B = x\sqrt{\frac{(1-b)}{(1+b)}} \tag{6}$$

$$t_B = t\sqrt{\frac{(1-b)}{(1+b)}} \tag{7}$$

To eliminate a common variable, x and t from (4) and (5) are substituted into (6) and (7) yielding

$$x_B = x_A\sqrt{\frac{(1+a)}{(1-a)}}\sqrt{\frac{(1-b)}{(1+b)}} \tag{8}$$

$$t_B = t_A\sqrt{\frac{(1+a)}{(1-a)}}\sqrt{\frac{(1-b)}{(1+b)}} \tag{9}$$

Simplifying the velocity terms in (8) yields

$$\sqrt{\frac{(1 - ab - (b - a))}{(1 - ab + (b - a))}} \qquad (10)$$

which is rewritten as

$$\sqrt{\frac{(1 - s)}{(1 + s)}} \qquad (11)$$

with

$$s = \frac{b - a}{1 - ab} \qquad (12)$$

the expression for calculating the relative velocity for A and B.

The x and t measurements of A and B are related via (8) and (9), as functions of relative velocity (12) and independently of U.