## 46 Binary Change

A "binary system" of acceleration may be represented as two vectors of acceleration (each 4D) acting upon a common point and diverging by an angle of action ("system angle"). The binary system may also be represented as a rank two field tensor or "system tensor", which is a product of the vectors.

The system changes, so that each vector changes direction. The magnitude of each vector remains unchanged. A space-time "change function" is represented by a 4D "change vector" (not Nabla). The system change is represented as the dot product of the change vector and the system tensor. The system change is not divergence. The result of the change is a new vector of acceleration, having "special characteristics" which contain information relating to the original system ("history").

The special characteristics are defined by "vector geometry". The resulting vector (4D) has a geometry which includes the intersection of three plane surfaces. Edges and angles relating to the surfaces retain the scalar magnitudes and the angle of action of the original system.

The Schwarzschild metric may be simply obtained from the resulting vector.

## The Binary System;

Two vectors $\left(\boldsymbol{g}_{1}, \boldsymbol{g}_{2}\right)$ represent acceleration. The two vectors may be associated with stress (pressure and shear). They act upon a common point having an "angle of action" ( $\beta$ ) between them, forming a "binary system". The vectors are:

$$
\begin{aligned}
& \boldsymbol{g}_{1}=\mathrm{g}_{11} \boldsymbol{e}_{11}+\mathrm{g}_{12} \boldsymbol{e}_{12}+\mathrm{g}_{13} \boldsymbol{e}_{13}+\mathrm{g}_{14} \boldsymbol{e}_{14} \\
& \boldsymbol{g}_{2}=\mathrm{g}_{21} \boldsymbol{e}_{21}+\mathrm{g}_{22} \boldsymbol{e}_{22}+\mathrm{g}_{23} \boldsymbol{e}_{23}+\mathrm{g}_{24} \boldsymbol{e}_{24}
\end{aligned}
$$

The binary system may be represented as a field tensor $\left(\boldsymbol{T}_{12}\right)$ : $\quad \boldsymbol{T}_{\mathbf{1 2}}=\boldsymbol{g}_{\boldsymbol{1}} \boldsymbol{g}_{\mathbf{2}}$

## Change;

Assume that both vectors $\left(\boldsymbol{g}_{1}, \boldsymbol{g}_{2}\right)$ have a change of direction, however the magnitudes remain unchanged.

A combined "space-time change operator" may be represented as a 4D "change vector" ( $\nabla_{0}$ ):

$$
\nabla_{0}=(\partial / \partial \mathrm{t}) \boldsymbol{e}_{01}+(\partial / \partial \mathrm{x}) \boldsymbol{e}_{02}+(\partial / \partial \mathrm{y}) \boldsymbol{e}_{03}+(\partial / \partial z) \boldsymbol{e}_{04}
$$

The change vector is not Nabla.

The "system change" is represented as a dot product of the change vector and the system tensor. System change will result in a new vector of acceleration $\left(\boldsymbol{g}_{3}\right)$ :

$$
\nabla_{0} \cdot T_{12}=g_{3}
$$

System change is not a divergence, and the result is not a scalar. The resulting vector ( $\boldsymbol{g}_{3}$ ) has special characteristics which relate to the magnitudes and angle of the original binary system. The resulting vector contains some "history" of the original vectors. The special characteristics are defined by "vector geometry". The resulting vector (in 4 dimensions) has a geometry which includes the intersection of three plane surfaces. Edges and angles relating to the surfaces retain the scalar magnitudes and the angle of action of the original system. The Schwarzschild metric may be simply obtained from the resulting vector.

## Vector Geometry;

A vector of acceleration $\left(g_{n}\right)$ is: $\quad g_{n}=g_{n 1} e_{n 1}+g_{n 2} e_{n 2}+g_{n 3} e_{n 3}+g_{n} e_{n 4}$

Where: $\quad n$ is a vector identifier $(n=1,2,3)$
$g_{n 1}, g_{n 2}, g_{n 3}, g_{n 4}$ are components of acceleration
$\boldsymbol{e}_{n 1}, \boldsymbol{e}_{n 2}, \boldsymbol{e}_{n 3}, \boldsymbol{e}_{n 4}$ are basis vectors (unit vectors)

The vector has a magnitude:

$$
\left|\boldsymbol{g}_{n}\right|=g_{n 5}
$$

The magnitude is related to components:

$$
g_{n 1}^{2}+g_{n 2}^{2}+g_{n 3}^{2}+g_{n 4}^{2}=g_{n 5}^{2}
$$

"Radial components" ( $\left.g_{n 1}, g_{n 5}, g_{n 6}, g_{n 7}\right)$ all connect to the "tail" of the vector (origin of basis vectors).

Where: $\quad \mathrm{gnn}^{2}=\mathrm{g}_{\mathrm{n} 5}{ }^{2}-\mathrm{g}_{\mathrm{n} 4}{ }^{2}=\mathrm{g}_{\mathrm{n}}{ }^{2}+\mathrm{gn}_{\mathrm{n}}{ }^{2}$
$\mathrm{gn}^{2}=\mathrm{gn}^{2}-\mathrm{g}_{\mathrm{n} 3}{ }^{2}=\mathrm{g}_{\mathrm{n} 1^{2}}+\mathrm{g}_{\mathrm{n} 2}{ }^{2}$

A vector (in 4 dimensions) has "vector geometry". It may be represented as the "connection" of three plane surfaces. Each surface is a right triangle having three edges:

Surface 1: $\quad\left(g_{n 1}, g_{n 2}, g_{n 7}\right)$

Surface 2: $\quad\left(g_{n 7}, g_{n 3}, g_{n 6}\right)$

Surface 3: $\quad\left(g_{n 6}, g_{n 4}, g_{n 5}\right)$
Vector geometry gives a total of seven edges. The surfaces are "connected" at two radial edges ( $g_{n 6}, g_{n 7}$ ). The four vector components $\left(g_{n 1}, g_{n 2}, g_{n 3}, g_{n 4}\right)$ are each represented as an edge. The length of one radial edge $\left(g_{n 5}\right)$ is the magnitude.

Each surface contains one right angle and two complimentary angles. One angle of each surface is designated as a "prime angle". Vector geometry gives three prime angles ( $A_{n 1}, A_{n 2}, A_{n 3}$ ) for each vector.

## Binary Change

Angular geometry is:

| Surface $\mathrm{n} 1:$ | $\mathrm{g}_{\mathrm{n} 1}=\mathrm{g}_{\mathrm{n} 7} \operatorname{Cos}\left(\mathrm{~A}_{\mathrm{n} 1}\right)$ | and: | $\mathrm{g}_{\mathrm{n} 2}=\mathrm{g}_{\mathrm{n} 7} \operatorname{Sin}\left(\mathrm{~A}_{\mathrm{n} 1}\right)$ |
| :--- | :--- | :--- | :--- |
| Surface $\mathrm{n} 2:$ | $\mathrm{g}_{\mathrm{n} 7}=\mathrm{g}_{\mathrm{n} 6} \operatorname{Cos}\left(\mathrm{~A}_{\mathrm{n} 2}\right)$ | and: | $\mathrm{g}_{\mathrm{n} 3}=\mathrm{g}_{\mathrm{n} 6} \operatorname{Sin}\left(\mathrm{~A}_{\mathrm{n} 2}\right)$ |
| Surface $\mathrm{n} 3:$ | $\mathrm{g}_{\mathrm{n} 6}=\mathrm{g}_{\mathrm{n} 5} \operatorname{Cos}\left(\mathrm{~A}_{\mathrm{n} 3}\right)$ | and: | $\mathrm{g}_{\mathrm{n} 4}=\mathrm{g}_{\mathrm{n} 5} \operatorname{Sin}\left(\mathrm{~A}_{\mathrm{n} 3}\right)$ |

## The Resultant Vector;

The result of system change is a new vector $\left(\boldsymbol{g}_{3}\right)$. The new vector is not a cross product of original vectors. Five "rules of change" apply to the new vector:

- $\quad$ The magnitude of one original vector is equal to the magnitude of $\boldsymbol{g}_{3}$ :
$g_{25}=g_{35}$
- The magnitude of one original vector is equal to a radial edge of $\boldsymbol{g}_{3}$ :
- The angle between original vectors is equal to a prime angle of $\boldsymbol{g}_{3}$ :
$g_{15}=g_{31}$
- The remaining prime angles of $\boldsymbol{g}_{3}$ are equal:
- A radial edge of $\boldsymbol{g}_{3}$ is related to the dot product of the original vectors:
$A_{31}=A_{32}$
$\boldsymbol{g}_{1} \cdot \boldsymbol{g}_{2}=\mathrm{g}_{37}{ }^{2}$

The five rules of system change relate the "history" of the original system to the "geometry" of $\boldsymbol{g}_{\mathbf{3}}$. The rules combine as follows:

$$
\begin{array}{ll}
A_{31}=A_{32} & \text { from rule: } 4 \\
\operatorname{Cos}\left(A_{31}\right)=\operatorname{Cos}\left(A_{32}\right) & \\
g_{31} / g_{37}=g_{37} / g_{36} & \text { from geometry } \\
g_{31} g_{36}=g_{37}{ }^{2} & \\
g_{31} g_{35} \operatorname{Cos}\left(A_{33}\right)=g_{37^{2}} & \text { from surface 33: } g_{36}=g_{35} \operatorname{Cos}\left(A_{33}\right) \\
g_{31} g_{35} \operatorname{Cos}(\beta)=g_{37^{2}} & \text { from rule: } 3 \\
g_{15} g_{25} \operatorname{Cos}(\beta)=g_{37^{2}} & \text { from rules: } 1,2 \\
g_{1} \cdot g_{2}=g_{37^{2}} & \text { giving rule:5 }
\end{array}
$$

Change Rule 5 may also be represented as a summation of products:

$$
g_{11} g_{21}+g_{12} g_{22}+g_{13} g_{23}+g_{14} g_{24}=g_{31} g_{36}
$$

## Binary Change

## Motion:

Assume two types of motion, wave motion and particle motion.

The "fundamental components" of particle motion ( $\mathrm{r}, \mathrm{t}, \mathrm{v}, \mathrm{u}$ ) are:
Spatial Distance (r) and: Time (t)
Particle Speed (v) and: displacement (u) (curved or straight paths)

The "fundamental components" of wave motion ( $R, T, C, \lambda$ ) are:

| Light Distance $(R)$ | and: | Light Time $(T)$ : inverse of frequency $(f)$ |
| :--- | :--- | :--- |
| Light Speed (c) | and: | Light Wavelength $(\lambda)$ |

## The Schwarzschild Metric:

Change Rule 5 relates components of acceleration. It may also be written as fundamental components of motion ( $r, t, v, u$ ) and ( $R, T, c, \lambda$ ), leading to the Schwarzschild metric. Change rule 5 is:

$$
g_{11} g_{21}+g_{12} g_{22}+g_{13} g_{23}+g_{14} g_{24}=g_{31} g_{36}
$$

Assume a definition of scalar acceleration $\left(g_{n N}\right)$ is:

$$
g_{n N}=v_{n N}{ }^{2} / r_{n N}
$$

Where: $\mathrm{v}_{\mathrm{nN}}$ is speed
$r_{n N}$ is distance

Substitution gives:

$$
\left(v_{11}^{2} / r_{11}\right)\left(v_{21}^{2} / r_{21}\right)+\left(v_{12}^{2} / r_{12}\right)\left(v_{22}^{2} / r_{22}\right)+\left(v_{13}^{2} / r_{13}\right)\left(v_{23}^{2} / r_{23}\right)+\left(v_{14}^{2} / r_{14}\right)\left(v_{24}^{2} / r_{24}\right)=\left(v_{31}^{2} / r_{31}\right)\left(v_{36}^{2} / r_{36}\right)
$$

Components of motion include light speed (c): $\quad c=v_{11}=v_{31}$
Giving: $\left(c^{2} / r_{11}\right)\left(v_{21}{ }^{2} / r_{21}\right)+\left(v_{12}{ }^{2} / r_{12}\right)\left(v_{22}{ }^{2} / r_{22}\right)+\left(v_{13}{ }^{2} / r_{13}\right)\left(v_{23}{ }^{2} / r_{23}\right)+\left(v_{14}{ }^{2} / r_{14}\right)\left(v_{24}{ }^{2} / r_{24}\right)=\left(c^{2} / r_{31}\right)\left(v_{36}{ }^{2} / r_{36}\right)$
Components of motion include spatial distance ( $r$ ) and light Distance ( $R$ ):

$$
\begin{aligned}
& r=r_{11}=r_{13}=r_{14}=r_{31}=r_{36} \\
& R=r_{21}=r_{12}=r_{22}=r_{23}=r_{24}
\end{aligned}
$$

Giving: $\left(c^{2} / r\right)\left(v_{21}{ }^{2} / R\right)+\left(v_{12}{ }^{2} / R\right)\left(v_{22}{ }^{2} / R\right)+\left(v_{13}{ }^{2} / r\right)\left(v_{23}{ }^{2} / R\right)+\left(v_{14}{ }^{2} / r\right)\left(v_{24}{ }^{2} / R\right)=\left(c^{2} / r\right)\left(v_{36}{ }^{2} / r\right)$
And: $\quad c^{2} v_{21}{ }^{2}+(r / R) v_{12}{ }^{2} v_{22}{ }^{2}+v_{13}{ }^{2} v_{23}{ }^{2}+v_{14}{ }^{2} v_{24}{ }^{2}=(R / r) c^{2} v_{36}{ }^{2}$
Assume a definition of scalar velocity $\left(v_{n N}\right)$ : $\quad v_{n N}=\partial u_{n N} / \partial t_{n N}$

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Where: $\mathrm{u}_{\mathrm{nN}}$ is displacement (curved or straight paths)
$\mathrm{t}_{\mathrm{nN}}$ is time
Giving: $c^{2}\left(\partial u_{21}{ }^{2} / \partial t_{21}{ }^{2}\right)+(r / R)\left(\partial u_{12}{ }^{2} / \partial t_{12}{ }^{2}\right)\left(\partial u_{22}{ }^{2} / \partial t_{22}{ }^{2}\right)+\left(\partial u_{13}{ }^{2} / \partial t_{13}{ }^{2}\right)\left(\partial u_{23}{ }^{2} / \partial t_{23}{ }^{2}\right)+$

$$
\left(\partial u_{14}{ }^{2} / \partial t_{14}{ }^{2}\right)\left(\partial u_{24}{ }^{2} / \partial t_{24}{ }^{2}\right)=(R / r) c^{2}\left(\partial u_{36}{ }^{2} / \partial t_{36}{ }^{2}\right)
$$

Components of motion include time $(t)$ and light time $(T)$, the inverse of frequency $(f)$ :

$$
\begin{aligned}
& \mathrm{t}=\mathrm{t}_{21}=\mathrm{t}_{22}=\mathrm{t}_{23}=\mathrm{t}_{24} \\
& \mathrm{~T}=\mathrm{t}_{12}=\mathrm{t}_{13}=\mathrm{t}_{14}=\mathrm{t}_{36}
\end{aligned}
$$

Giving: $c^{2}\left(\partial u_{21}{ }^{2} / \partial t^{2}\right)+(r / R)\left(\partial u_{12}{ }^{2} / \partial T^{2}\right)\left(\partial u_{22}{ }^{2} / \partial t^{2}\right)+\left(\partial u_{13}{ }^{2} / \partial T^{2}\right)\left(\partial u_{23}{ }^{2} / \partial t^{2}\right)+$

$$
\left(\partial u_{14}{ }^{2} / \partial T^{2}\right)\left(\partial u_{24}{ }^{2} / \partial t^{2}\right)=(R / r) c^{2}\left(\partial u_{36}{ }^{2} / \partial T^{2}\right)
$$

And: $\quad c^{2} \partial u_{21}{ }^{2} \partial T^{2}+(r / R) \partial u_{12}{ }^{2} \partial u_{22}{ }^{2}+\partial u_{13}{ }^{2} \partial u_{23}{ }^{2}+\partial u_{14}{ }^{2} \partial u_{24}{ }^{2}=(R / r) c^{2} \partial u_{36}{ }^{2} \partial t^{2}$
Components of motion include wavelength ( $\lambda$ ):

$$
\lambda=u_{21}=u_{22}=u_{23}=u_{24}=u_{36}
$$

Giving: $c^{2} \partial \lambda^{2} \partial T^{2}+(r / R) \partial u_{12}{ }^{2} \partial \lambda^{2}+\partial u_{13}{ }^{2} \partial \lambda^{2}+\partial u_{14}{ }^{2} \partial \lambda^{2}=(R / r) c^{2} \partial \lambda^{2} \partial t^{2}$
and: $\quad c^{2} \partial T^{2}+(r / R) \partial u_{12}{ }^{2}+\partial u_{13}{ }^{2}+\partial u_{14}{ }^{2}=(R / r) c^{2} \partial t^{2}$
Curved path displacement ( $u_{13}, u_{14}$ ) for particle motion is:

$$
\begin{array}{llll}
u_{13}=r \theta & \text { and: } & u_{14}=w \phi \\
\partial u_{13}=r \partial \theta & \text { and: } & \partial u_{14}=w \partial \phi \quad \text { and: } \quad w=r \operatorname{Sin}(\theta)
\end{array}
$$

giving: $c^{2} \partial T^{2}+(r / R) \partial u_{12}{ }^{2}+r^{2} \partial \theta^{2}+r^{2} \operatorname{Sin}^{2}(\theta) \partial \phi^{2}=(R / r) c^{2} \partial t^{2}$
Incremental straight path displacement $\left(\partial u_{12}\right)$ is: $\partial u_{12}=\partial r$
Giving the Schwarzschild metric:

$$
c^{2} \partial T^{2}+(r / R) \partial r^{2}+r^{2} \partial \theta^{2}+r^{2} \operatorname{Sin}^{2}(\theta) \partial \phi^{2}=(R / r) c^{2} \partial t^{2}
$$

Where: $R=r-r_{s}$
$r_{s}$ is the Schwarzschild radius
( $\mathrm{t}, \mathrm{r}, \theta, \phi$ ) are the dimensions of particle motion
( $T, R, \theta_{\lambda}, \phi_{\lambda}$ ) are the dimensions of wave motion (light motion)

Assume curved path displacement ( $\mathrm{u}_{23}, \mathrm{u}_{24}$ ) for wave motion is:

$$
\begin{array}{lll}
\mathrm{u}_{23}=\lambda=\mathrm{R} \theta_{\lambda} & \text { and: } \quad \mathrm{u}_{24}=\lambda=\mathrm{W} \phi_{\lambda} \\
\partial \mathrm{u}_{23}=\mathrm{R} \partial \theta_{\lambda} & \text { and: } & \partial \mathrm{u}_{24}=\mathrm{W} \partial \phi_{\lambda} \quad \text { and: } \quad W=R \operatorname{Sin}\left(\theta_{\lambda}\right)
\end{array}
$$

## Conclusion:

A binary system of acceleration may be represented as a "system tensor" (field tensor). Change of space-time may be represented as an operator, the "change vector". A system change may be represented as a dot product of the change vector and the system tensor (not divergence). The result of change is a field vector which contains the "history" of the system. The system history is defined by five rules of change relating to vector geometry.

Two types of motion may be assumed: "particle motion" and "wave motion". Fundamental characteristics are associated with each type of motion.

One rule of change may be represented using the fundamental characteristics of both types of motion, giving the Schwarzschild metric.

