08 Particle Interaction

An interaction between two particles is assumed to be mediated by an "exchange" of bosons. A force may be associated with each boson and with each particle.

Force may be represented as a vector. The interaction may be represented as a four vector set. The four vectors may be arranged as two pairs of vectors, each pair represents forces associated with a particle and a "corresponding boson".

The vector pairs must be "compatible" for energy transfer so that compatibility conditions apply to the vectors. An interaction normally "conserves" energy and momentum.

An interaction of special interest is between an electron and a photon as represented by the Compton equation, published in 1923. Energy is transferred from the photon to the electron. The collision is elastic and each particle recoils in a different direction. Vector components will give the Compton equation.

The Force Vector;

Four vectors of force ($\boldsymbol{F_1}$, $\boldsymbol{F_2}$, $\boldsymbol{F_3}$, $\boldsymbol{F_4}$) are required. Generally a 3D vector of force ($\boldsymbol{F_n}$) is;

$$\boldsymbol{F_n} = F_{n1}\boldsymbol{e_{n1}} + F_{n2}\boldsymbol{e_{n2}} + F_{n3}\boldsymbol{e_{n3}}$$

Where; 'n' is a vector identifier (n = 1,2,3,4)

$\boldsymbol{e_{n1}}$, $\boldsymbol{e_{n2}}$, $\boldsymbol{e_{n3}}$ are basis vectors (orthogonal unit vectors)

F_{n1} , F_{n2} , F_{n3} are components of force

Each vector has a magnitude; $|\boldsymbol{F_n}| = F_{n4}$

The scalar components are related to the magnitude; $F_{n1}^2 + F_{n2}^2 + F_{n3}^2 = F_{n4}^2$

A sub-component of force (F_{n5}) is; $F_{n5}^2 = F_{n1}^2 + F_{n2}^2 = F_{n4}^2 - F_{n3}^2$

Angular Geometry;

Each vector has "component angles" (A_{n1}, A_{n2}) having geometry;

$F_{n1} = F_{n5}\text{Cos}(A_{n1})$ and $F_{n2} = F_{n5}\text{Sin}(A_{n1})$

$F_{n5} = F_{n4}\text{Cos}(A_{n2})$ and $F_{n3} = F_{n4}\text{Sin}(A_{n2})$

Vector Pairs;

The four vectors of force may be grouped as two vector pairs ($\boldsymbol{F_1}$, $\boldsymbol{F_3}$)($\boldsymbol{F_2}$, $\boldsymbol{F_4}$). These forces are associated with two interacting particles (P_1, P_2) and two bosons (B_3, B_4).

Where; B_3 is the "corresponding boson" to P_1 giving the associated "force pair" ($\boldsymbol{F_1}$, $\boldsymbol{F_3}$)

B_4 is the "corresponding boson" to P_2 giving the associated "force pair" ($\boldsymbol{F_2}$, $\boldsymbol{F_4}$)

A "vector pair" shares; a common origin

a common angular geometry $(A_{11} = A_{31})(A_{12} = A_{32})$

$(A_{21} = A_{41})(A_{22} = A_{42})$

a common frame of reference $(\boldsymbol{e_{11}}, \boldsymbol{e_{12}}, \boldsymbol{e_{13}}) = (\boldsymbol{e_{31}}, \boldsymbol{e_{32}}, \boldsymbol{e_{33}})$

$(\boldsymbol{e_{21}}, \boldsymbol{e_{22}}, \boldsymbol{e_{23}}) = (\boldsymbol{e_{41}}, \boldsymbol{e_{42}}, \boldsymbol{e_{43}})$

Compatibility;

All vectors must be compatible for interaction. The six compatibility rules are;

$A_{11} = A_{21}$	$F_{11} = F_{43}$	$F_{21} = F_{33}$
$F_{35} = F_{15} + F_{22}$	$F_{45} = F_{12} + F_{25}$	$F_{34} = F_{44}$

From compatibility rule 1; $A_{11} = A_{21}$

$\mathrm{Sin}(A_{11}) = \mathrm{Sin}(A_{21})$

$F_{12}/F_{15} = F_{22}/F_{25}$

Conservation;

A component of force (F_{nN}) is related to a component of momentum (p_{nN}); $F_{nN} = p_{nN}/t_0$

Where; t_0 is a time scalar

The conservation rule for momentum is; $p_{35} = p_{45}$

From compatibility rules 4, and 5; $p_{15} + p_{22} = p_{12} + p_{25}$

Where; p_{35} is total initial momentum

p_{45} is total final momentum

p_{15} is the initial momentum of particle 1

p_{22} is the initial momentum of particle 2

p_{12} is the final momentum of particle 1

p_{25} is the final momentum of particle 2

The conservation rule for momentum may be written as;

$$p_{15} - p_{12} = p_{25} - p_{22}$$

$$1 - p_{12}/p_{15} = p_{22}[p_{25}/(p_{22}p_{15}) - 1/p_{15}]$$

From compatibility rule 1; $1 - p_{12}/p_{15} = p_{22}[1/p_{12} - 1/p_{15}]$

From angular geometry; $1 - \text{Sin}(A_{11}) = p_{22}[1/p_{12} - 1/p_{15}]$

A scattering angle (θ) is; $\theta = ½\pi - A_{11}$

Giving; $\text{Sin}(A_{11}) = \text{Cos}(\theta)$

The conservation rule for momentum may be written as;

$$1 - \text{Cos}(\theta) = p_{22}[1/p_{12} - 1/p_{15}]$$

The Compton Equation;

An interaction of special interest is between an electron and a photon as represented by the Compton equation, published in 1923. Energy is transferred from the photon to the electron. The collision is elastic and each particle recoils in a different direction. Vector components of momentum will relate the conservation rule to the Compton equation.

Where; p_{1N} is the photon

p_{2N} is the electron

Components of momentum are defined as;

$p_{15} = h/\lambda_{15}$ (initial momentum of photon)

$p_{12} = h/\lambda_{12}$ (final momentum of photon)

$p_{22} = m_e c$ (initial momentum of electron)

p_{25} (final momentum of electron)

The conservation rule for momentum is;

$$1 - \text{Cos}(\theta) = p_{22}[1/p_{12} - 1/p_{15}]$$

Substituting for momentum gives;

$$(1 - Cos_\theta) = m_e c[\lambda_{12}/h - \lambda_{15}/h]$$

The Compton equation may be written as;

$$\lambda_{12} - \lambda_{15} = (1 - Cos_\theta)(h/m_e c)$$

Where; θ is the scattering angle

λ_{15} is the initial wavelength of the photon

λ_{12} is the wavelength of photon after scattering ($\lambda_{12} > \lambda_{15}$)

m_e is electron rest mass

h is the Plank constant

c is the light constant

Conclusion;

The Compton Effect is a unique interaction which may be represented by four vectors of momentum. Each vector represents a momentum. Vector components give the Compton equation.