# 47 The Geometry of Force

Force may be represented as a vector. A 4D vector of force has four components, it also has four “associated components” (including magnitude), giving eight “parts” of force.

The parts of a 4D vector may be arranged to form five “connected plane surfaces”. Each surface is a right triangle and is associated with a “primary angle”. The primary angles are related to vector parts as “vector geometry”. This gives a vector of force having a “geometry of force”.

If one angular condition is true, then relationships between various parts and primary angles may be represented as “scalar equations of force”.

Each part of force may be associated with an energy. If definitions of force and energy apply, then seven “systems of energy” may be represented. The seven systems are:

- Klein-Gordon energies (wave-particle energies)
- Stefan-Boltzmann energies (radiant energies)
- Bohr energies (orbital energies)
- Compton energies (photo-electric energies)
- energies of mass dilation
- Plank energies (thermo-gravitational energies)
- Hawking energies (photo-thermo-gravitational energies)

## *The Force Vector;*

Force may be represented as a 4D vector ($\boldsymbol{F}$): $\boldsymbol{F} = F_1\boldsymbol{e_1} + F_2\boldsymbol{e_2} + F_3\boldsymbol{e_3} + F_4\boldsymbol{e_4}$

Where: $F_1$ , $F_2$ , $F_3$ , $F_4$ are components of force (scalars)

$\boldsymbol{e_1}$ , $\boldsymbol{e_2}$ , $\boldsymbol{e_3}$ , $\boldsymbol{e_4}$ are basis vectors (unit vectors)

## *Vector Parts;*

The “vector parts” include the all the components, also a magnitude, and “sub-components”.

The force vector has a magnitude: $|\boldsymbol{F}| = F_5$

The magnitude is related to components: $F_5^2 = F_1^2 + F_2^2 + F_3^2 + F_4^2$

The sub-components ($F_6$ , $F_7$ , $F_8$) are: $F_6^2 = F_7^2 + F_3^2 = F_5^2 - F_4^2$

$F_7^2 = F_1^2 + F_2^2 = F_6^2 - F_3^2$

$F_8^2 = F_2^2 + F_3^2 = F_6^2 - F_1^2$

## *Angular Geometry;*

The force vector has five plane "surfaces". Each surface is a right triangle and includes a "primary angle" ($A_1$ , $A_2$ , $A_3$ , $A_4$ , $A_5$). Each primary angle "represents" a plane surface. The angles are related to vector parts as "angular geometry":

$F_1 = F_7Cos(A_1)$ $\qquad$ $F_2 = F_7Sin(A_1)$

$F_7 = F_6Cos(A_2)$ $\qquad$ $F_3 = F_6Sin(A_2)$

$F_6 = F_5Cos(A_3)$ $\qquad$ $F_4 = F_5Sin(A_3)$

$F_1 = F_6Cos(A_4)$ $\qquad$ $F_8 = F_6Sin(A_4)$

$F_2 = F_8Cos(A_5)$ $\qquad$ $F_3 = F_8Sin(A_5)$

## *The Force Equations;*

Assume one restriction: $A_2 = A_3$

Giving: $Cos(A_2) = Cos(A_3)$

From angular geometry: $F_7/F_6 = F_6/F_5$

Giving: $F_5F_7 = F_6^2$

From angular geometry: $F_1 = F_6Cos(A_4)$ and: $F_8 = F_6Sin(A_4)$

$F_1^2 = F_6^2C^2_{A4}$ and: $F_8^2 = F_6^2S^2_{A4}$

The force equations are: $F_1^2 = F_5F_7C^2_{A4}$ and: $F_8^2 = F_5F_7S^2_{A4}$

## *Klein-Gordon Energies;*

The general force equation is: $F_1^2 = F_5F_7C^2_{A4}$

$F_1^2 = F_6^2C^2_{A4}$

Assume: $S_{A4} = v/c$

$C^2_{A4} = 1 - S^2_{A4} = 1 - v^2/c^2$

Giving: $F_1^2 = F_6^2(1 - v^2/c^2)$

Where: v is velocity

c is the light constant

Assume force definitions: $F_1 = E_1/\lambda$ and: $F_6 = E_6/\lambda$

Assume energy definitions: $E_1 = m_0vc$ and: $E_6 = ħf$

Where: $m_0$ is rest mass

*f* is frequency

λ is wavelength

ħ is the reduced Plank constant

Giving squares of force: $(m_0vc/\lambda)^2 = (ħf/\lambda)^2(1 - v^2/c^2)$

Giving squares of energy: $(m_0vc)^2 = (ħf)^2(1 - v^2/c^2)$

$(v^2/c^2)(ħf)^2 + (m_0vc)^2 = (ħf)^2$

$ħ^2f^2 + m_0^2c^4 = c^2ħ^2(f^2/v^2)$

A frequency operator is: $\partial^2/\partial t^2 = f^2$

A distance operator is: $\partial^2/\partial R^2$

A ratio of operators is: $(\partial^2/\partial t^2)/(\partial^2/\partial R^2) = \partial R^2/\partial t^2 = v^2$

A "ratio of motion" is: $f^2/v^2 = (\partial^2/\partial t^2)(\partial^2/\partial R^2)/(\partial^2/\partial t^2) = \partial^2/\partial R^2$

Squares of energy are: $ħ^2f^2 + m_0^2c^4 = c^2ħ^2(f^2/v^2)$

Operator substitution: $ħ^2(\partial^2/\partial t^2) + m_0^2c^4 = c^2ħ^2(\partial^2/\partial R^2)$

A wave function (ψ) interacts with the energy squares: $ħ^2(\partial^2\psi/\partial t^2) + m_0^2c^4\psi = c^2ħ^2(\partial^2\psi/\partial R^2)$

The Klein-Gordon equation may be written as: $0 = (1/c^2)(\partial^2\psi/\partial t^2) - \partial^2\psi/\partial R^2 + (m_0^2c^2/ħ^2)\psi$

A wavelength operator is: $\partial^2/\partial\lambda^2 = \partial^2/c^2\partial t^2$ giving: $c^2 = \partial\lambda^2/\partial t^2 = \lambda^2f^2$

A "combined operator" ($\underline{\boldsymbol{H}}^2$) includes both wavelength and distance operators: $\underline{\boldsymbol{H}}^2 = ħ^2c^2(\partial^2/\partial\lambda^2 - \partial^2/\partial R^2)$

The Klein-Gordon energy squares may be written as: $\underline{\boldsymbol{H}}^2 = -m_0^2c^4$

The "Klein-Gordon energy" is: $\underline{\boldsymbol{H}} = im_0c^2$

The Klein-Gordon "root equation" is: $\underline{\boldsymbol{H}}\psi = i\psi m_0c^2$

## *Stefan-Boltzmann Energies;*

The general force equation is: $F_1^2 = F_5F_7C^2_{A4}$

Assume force definitions: $F_1 = E_1^2/cħ$ and: $F_5 = E_5/r$ and: $F_7 = E_7/r$

Where: $\hbar$ is the reduced Plank constant

Giving forces: $E_1^4/c^2\hbar^2 = (E_5/r)(E_7/r)C^2_{A4}$

Assume energy definitions: $E_1$ is thermal energy of emitter: $E_1 = ½\pi kT$

(where; k is the Boltzmann constant)

$E_5$ is total energy of emitter: $E_5 = ½hf$

$E_7$ is radiant energy

Giving an equation of forces: $(½\pi kT)^4/c^2\hbar^2 = (½hf/r)(E_7/r)C^2_{A4}$

$4\pi^2(½\pi kT)^4/c^2h^2 = (½hf/r)(E_7/r)C^2_{A4}$

$8\pi^2(½\pi kT)^4/c^2h^3 = (fE_7/r^2)C^2_{A4}$

$2\pi(½\pi kT)^4/c^2h^3 = (fE_7/4\pi r^2)C^2_{A4}$

Power ($P_7$) of radiation: $P_7 = fE_7$

Emissive surface area ($A_7$) is: $A_7 = 4\pi r^2$ (where; r is the radius of the emitter)

Giving: $2\pi(½\pi kT)^4/c^2h^3 = (P_7/A_7)C^2_{A4}$

Brightness ($J_7$) is power per emissive area: $J_7 = P_7/A_7$

Giving: $2\pi(½\pi kT)^4/c^2h^3 = J_7C^2_{A4}$

$2\pi^5k^4T^4/(16c^2h^3) = J_7C^2_{A4}$

$2\pi^5k^4T^4/c^2h^3 = 16J_7C^2_{A4}$

Assume: $S_{A4} = v/c$

$C^2_{A4} = 1 - S^2_{A4} = 1 - v^2/c^2$

Where: v is average velocity associated with thermal energy

c is the light constant

Giving: $2\pi^5k^4T^4/c^2h^3 = 16J_7(1 - v^2/c^2)$

assume thermal velocity: $v = ¼c$

Giving: $2\pi^5k^4T^4/c^2h^3 = J_7(16 - 1)$

$2\pi^5k^4T^4/15c^2h^3 = J_7 = \sigma T^4$

Where: σ is the Stefan-Boltzmann constant: $\sigma = 2\pi^5k^4/15c^2h^3$

## *Bohr Energies;*

Bohr energies are restricted to the hydrogen atom or to ions with a single electron.

Bohr forces require that two angular conditions apply to the force vector:

| | | | |
|---|---|---|---|
| | $A_1 = A_2$ | and: | $A_2 = A_3$ |
| Giving: | $\cos(A_1) = \cos(A_2)$ | and: | $\cos(A_2) = \cos(A_3)$ |
| From angular geometry: | $F_1/F_7 = F_7/F_6$ | and: | $F_7/F_6 = F_6/F_5$ |
| Giving: | $F_1F_6 = F_7^2$ | and: | $F_5F_7 = F_6^2$ |

The combined conditions give: $F_6^2 = F_7^4/F_1^2 = F_5F_7$

Giving the Bohr force equation: $F_7^3 = F_5F_1^2$

$F_7^3 = F_5F_6^2C^2_{A4}$

Assume force definitions: $F_7 = E_7/\lambda$ and: $F_5 = E_5/\lambda$ and: $F_6 = E_6/\lambda$

Giving forces: $(E_7/\lambda)^3 = (E_5/\lambda)(E_6^2/\lambda^2)C^2_{A4}$

Giving energies: $E_7^3 = E_5E_6^2C^2_{A4}$

Assume energy definitions: $E_5$ is massive energy: $E_5 = ½m_ec^2$

$E_7$ is emissive energy: $E_7 = hf$

$E_6$ is electric energy: $E_6 = k_eq_1q_2/r$

Where: $m_e$ is the mass of the electron

$k_e$ is the electric field constant: $k_e = 1/(4\pi\varepsilon_0)$

$q_1$ is the charge of an electron; $q_1 = -e$

$q_2$ is the charge of the nucleus: $q_2 = Ze$

r is the average radial distance between the nucleus and the electron

The Bohr energies are: $E_7^3 = E_5E_6^2C^2_{A4}$

$(hf)^3 = (½m_ec^2)(k_eq_1q_2/r)^2C^2_{A4}$

$(hf)^3 = (½m_ec^2)(Z^2e^4/16\pi^2\varepsilon_0^2r^2)C^2_{A4}$

$$(hf)(hc/\lambda)^2 = (\tfrac{1}{2}m_ec^2)(Z^2e^4/4\varepsilon_0^2)(1/4\pi^2r^2)C^2_{A4}$$

$$(hf)(h^2c^2/\lambda^2) = (m_ec^2Z^2e^4/8\varepsilon_0^2)(1/4\pi^2r^2)C^2_{A4}$$

$$(hf)(h^2/\lambda^2) = (m_eZ^2e^4/8\varepsilon_0^2)(1/4\pi^2r^2)C^2_{A4}$$

Assume: $S^2_{A4} = v_2/v_1$ giving: $C^2_{A4} = 1- v_2^2/v_1^2$ (where; $v_1$ , $v_2$ are initial and final velocities)

The Bohr energies become: $(hf)(h^2/\lambda^2) = (m_eZ^2e^4/8\varepsilon_0^2)(1/4\pi^2r^2)(1- v_2^2/v_1^2)$

Quantization rules are: Rule 1: $n_1v_1= c$

Rule 2: $n_2v_2= c$

Rule 3: $2\pi r = n_1\lambda$

Where: $n_1$, $n_2$ are integers

The Bohr energies become: $(hf)(h^2/\lambda^2) = (m_eZ^2e^4/8\varepsilon_0^2)(1/n_1^2\lambda^2)(1- n_1^2/n_2^2)$

$$hf = (m_eZ^2e^4/8\varepsilon_0^2h^2)(1/n_1^2 - 1/n_2^2)$$

$$hc/\lambda = (m_eZ^2e^4/8\varepsilon_0^2h^2)(1/n_1^2 - 1/n_2^2)$$

In the case of hydrogen: $Z = 1$

Giving: $1/\lambda = (m_ee^4/8\varepsilon_0^2h^3c)(1/n_1^2 - 1/n_2^2) = R_H(1/n_1^2 - 1/n_2^2)$

Where: $R_H$ is the Rydberg constant: $R_H = m_ee^4/8\varepsilon_0^2h^3c$

## *Compton Energies:*

The Compton equation represents photo-electric interaction. This interaction may be represented as a vector of force. Parts of force give equation:

$$F_8 = F_6\mathrm{Sin}(A_4)$$

$$F_8^2 = F_5F_7S^2_{A4}$$

"Compton force" requires: $F_8 = F_7$

Which also implies: $F_1 = F_3$ and: $A_1 + A_5 = \tfrac{1}{2}\pi$

Giving: $F_7 = F_5S^2_{A4}$

$$F_7\lambda= (F_5\lambda)S^2_{A4}$$

Giving energies: $E_7 = E_5S^2_{A4}$

Assume energy definitions: $E_7$ is massive energy: $E_7 = ½m_ec^2$

$E_5$ is photonic energy: $E_5 = hc/\lambda_5$

Giving: $½m_ec^2 = (hc/\lambda_5)S^2_{A4}$

$m_ec = (4\pi\hbar/\lambda_5)S^2_{A4}$

Assume: $A_4 = ½\theta$ Where: $\theta$ is the scattering angle

Giving the Compton equation: $\lambda_5 = (4\pi\hbar/m_ec)S^2_{½\theta}$

Where: $\lambda_5 = \lambda_2 - \lambda_1$

## *Mass Dilation:*

From force geometry: $F_6^2 = F_1^2 + F_8^2$

$1 = (F_1/F_6)^2 + (F_8/F_6)^2$

Assume force definitions: $F_1 = m_0c^2/\lambda$

$F_8 = mvc/\lambda = hc/\lambda^2$ (from De Broglie momentum)

$F_6 = mc^2/\lambda$

giving mass dilation: $1 = (m_0/m)^2 + (v/c)^2$

Where: $m_0$ is rest mass

m is dynamic mass

v is scalar velocity

c is the light constant

## *Plank Energies:*

Thermo-gravitational energies may be obtained from the force vector.

A force equation is: $F_1^2 = F_5F_7C^2_{A4}$

Assume: $S_{A4} = v/c$ giving: $C^2_{A4} = 1- v^2/c^2$ (where; v is velocity)

The force equation becomes: $F_1^2 = F_5F_7(1- v^2/c^2)$

Assume force definitions: $F_1 = E_1/r$ $F_5 = E_5^2/hc$ $F_7 = E_7/r$

Assume energy definitions: $E_1 = m_1c^2$ $E_5 = k_BT_P$ $E_7 = Gm_7^2/r$

Giving: $(m_1c^2/r)^2 = (k_B{}^2T_P{}^2/hc)(Gm_7{}^2/r^2)(1- v^2/c^2)$

$(m_1c^2)^2 = (k_B{}^2T_P{}^2/hc)Gm_7{}^2(1- v^2/c^2)$

Mass dilation gives: $m_1{}^2c^4 = (k_B{}^2T_P{}^2/hc)Gm_1{}^2$

Giving Plank temperature: $T_P{}^2 = hc^5/Gk_B{}^2$

## *Hawking Energies:*

Photo-thermo-gravitational energies may be obtained from the force vector.

A force equation is: $F_1{}^2 = F_5F_7C^2{}_{A4}$

Assume force definitions: $F_1 = E_1/\lambda$ $F_5 = (v/c)E_5/\lambda$ $F_7 = E_7/\lambda$

Assume energy definitions: $E_1 = ½\hbar f$ $E_5 = k_BT_H$ $E_7 = Gm_7{}^2/\lambda$

Giving forces: $(½\hbar f/\lambda)^2 = (v/c)(k_BT_H/\lambda)(Gm_7{}^2/\lambda^2)(1- v^2/c^2)$

$(\hbar f/\lambda)^2 = 4(v/c)(k_BT_H/\lambda)(Gm_7{}^2/\lambda^2)(1- v^2/c^2)$

Mass dilation gives: $(\hbar f/\lambda)^2 = 4(v/c)(k_BT_H/\lambda)(Gm_0{}^2/\lambda^2)$

Where: $m_0{}^2 = m_7{}^2(1- v^2/c^2)$

$(hc/2\pi\lambda^2)^2 = 4(v/c)(k_BT_H/\lambda)(Gm_0{}^2/\lambda^2)$

De Broglie momentum gives: $(m_0vc/2\pi\lambda)(hc/2\pi\lambda^2) = 4(v/c)(k_BT_H/\lambda)(Gm_0{}^2/\lambda^2)$

$(m_0c^2/\lambda)(\hbar c/\lambda^2) = 8\pi(k_BT_H/\lambda)(Gm_0{}^2/\lambda^2)$

$(m_0c^2)(\hbar c) = 8\pi(k_BT_H)(Gm_0{}^2)$

$\hbar c^3 = 8\pi k_BT_HGm_0$

Hawking temperature is: $T_H = \hbar c^3/8\pi k_BGm_0$

## *Conclusion:*

A vector of force has a “geometry of force”. If one angular condition is true, then relationships between various “parts of force” and primary angles may be represented as “scalar equations of force”.

Each part of force may be associated with an energy. If definitions of force and energy apply, then seven “equations of energy” may be represented.