## 42 Particle Interaction

Two particles ("primary particles") interact. Energy is exchanged between them. The energy temporarily appears as a third particle, a "mediating particle".

The energies associated with the interaction are related to the energies of the Dirac equation, the relativistic Schrodinger equation, the Klein-Gordon equation, and the Compton equation.

The interaction is controlled by forces. Forces may be associated with each particle. Two types of force are required to complete the interaction: "exchange forces" and "primary forces". Primary forces are associated with motion of the primary particles. The mediating particle is not subject to primary forces. The "system of interaction" has three particles and ten forces (six exchange and four primary).

Each force may be represented as a vector. Components of the force vectors are considered to "align" and "combine" by "connection", "opposition" and "congruency", forming a "vector structure". The arrangement of the structure of particles and vectors is represented by an "Interaction Diagram".


Each particle exchanges force directly with each other particle. This means that each particle has two "associated exchange forces", giving a total of six exchange forces. "Exchange force pairs" link any two particles and may represent bosonic exchange. The result is a "convergence" of four vectors (two exchange pairs) at each particle.

A primary force is associated with the motion of a primary particle. Each primary particle has two primary forces, giving a total of four primary forces. "Prime pairs" always act upon the same particle, in different directions, connected (head to tail) at the center of the particle, with unequal strength. The mediating particle is not subject to prime forces.

Each circle on the Interaction Diagram represents a particle. Each line represents one primary force vector or two exchange force vectors (opposite directions).

A "reciprocating force pair" is represented as two force vectors of undefined force types, acting upon the same particle, connected (tail to tail) at the center of the particle, directed in opposition, having equal strength.
"Special reciprocation" is similar and requires defined force types and one "angular condition". The structure includes a "special reciprocating force pair" with different force types, so that one exchange force "specially reciprocates" with a primary force. The complex number (i) is assumed to represent a "component ratio of special reciprocity".

## Particles;

Two "primary particles" $\left(\mathrm{P}_{1}, \mathrm{P}_{2}\right)$ interact. Energy is exchanged between them. The energy appears temporarily as a third particle $\left(P_{3}\right)$, which is a "mediating particle".

## Forces;

The interaction is controlled by ten forces. Two types of force are required to complete the interaction: "exchange force" and "primary force".

Each force may be represented as a vector in space. Time will be represented by frequencies associated with the forces. The frequencies are functions of operators. This permits the use of 3D vectors (replacing the normal 4D vectors). All vectors associated with the same particle have a common origin (the center of the particle). The vectors may be considered to represent "instantaneous force".

Each particle has a direct interaction with each other particle, represented as an "exchange force vector". This means that each particle has two "associated" exchange forces, giving a total of six exchange vectors. Four exchange vectors (two vector pairs) "converge" at each particle.

The prime particles also have two prime forces each, giving a total of four prime vectors. Prime forces may be associated with the motion of a particle.

Two forces are reciprocal and are represented by "reciprocal vectors". A definition of "special reciprocation" will give a complex ratio of "reciprocal components". The complex number (i) is assumed to represent a "ratio of special reciprocity".

Force is represented as a vector $\left(F_{n}\right)$.
Where: n is a vector identifier $\quad(\mathrm{n}=0 \ldots 9)$

The exchange force vector directed from $P_{1}$ to $P_{3}$ is: $\quad F_{1}$

The exchange force vector directed from $P_{1}$ to $P_{2}$ is: $\quad F_{\mathbf{2}}$

The exchange force vector directed from $P_{3}$ to $P_{2}$ is: $\quad F_{3}$

The exchange force vector directed from $P_{2}$ to $P_{3}$ is: $\quad \boldsymbol{F}_{\mathbf{4}}$

The exchange force vector directed from $\mathrm{P}_{2}$ to $\mathrm{P}_{1}$ is: $\quad \boldsymbol{F}_{5}$

The exchange force vector directed from $\mathrm{P}_{3}$ to $\mathrm{P}_{1}$ is: $\boldsymbol{F}_{6}$
The prime force vectors associated with $P_{1}$ are: $\quad \boldsymbol{F}_{\mathbf{7}}, \boldsymbol{F}_{\mathbf{8}}$
The prime force vectors associated with $\mathrm{P}_{2}$ are: $\quad \boldsymbol{F}_{\mathbf{9}}, \boldsymbol{F}_{\mathbf{0}}$

## Exchange Pairs;

The vectors may also be grouped as "exchange pairs". Exchange pairs have equal magnitude and opposite direction however they are not "reciprocating" because they originate at different particles. Exchange pairs may be associated with bosonic exchange. The exchange pairs are:

The vectors $\left(\boldsymbol{F}_{1}, \boldsymbol{F}_{6}\right)$ are an "exchange pair" between particles $\left(\mathrm{P}_{1}, \mathrm{P}_{3}\right)$.

The vectors $\left(F_{\mathbf{3}}, \boldsymbol{F}_{\mathbf{4}}\right)$ are an "exchange pair" between particles $\left(\mathrm{P}_{3}, \mathrm{P}_{2}\right)$.

The vectors $\left(F_{5}, F_{2}\right)$ are an "exchange pair" between particles $\left(\mathrm{P}_{2}, \mathrm{P}_{1}\right)$.

If two force vectors $\left(\boldsymbol{F}_{a}, \boldsymbol{F}_{b}\right)$ are an exchange pair, then; $\mathrm{a}+\mathrm{b}=7$

## Vector Circuits;

Three exchange vectors $\left(\boldsymbol{F}_{\mathbf{1}}, \boldsymbol{F}_{3}, \boldsymbol{F}_{5}\right)$ act as a triangular "circuit of force" in space. The vector directions give a circuit directed as "clockwise". The exchange vectors "connect" (head to tail) at the centers of particles.

The other three exchange vectors $\left(\boldsymbol{F}_{2}, \boldsymbol{F}_{4}, \boldsymbol{F}_{6}\right)$ also act as a triangular "circuit of force" in space. The vector directions give a circuit directed as "counter-clockwise".

The circuits of force are congruent in space having opposite directions.

## The Interaction Structure;

The ten vectors of force are "connected" (head or tail) at the particle centers. They are also "aligned" in space by opposition (including reciprocation) and congruency, forming a "vector structure". Congruent components are spatially co-incident. The particle-force structure represents the total interaction and is represented by an "Interaction Diagram".

A prime vector $\left(\boldsymbol{F}_{8}\right)$ is reciprocal to the exchange vector $\left(\boldsymbol{F}_{1}\right)$.
Twenty "component rules" define the structure, they include rules of reciprocation, rules of congruency, and rules of non-congruency.

## Force Vectors;

A 3D force vector ( $\boldsymbol{F}_{n}$ ) may represent an "instantaneous force". The origin (tail) of a vector corresponds to the center of a particle. The vectors are:

$$
F_{n}=F_{n 1} e_{n 1}+F_{n 2} e_{n 2}+F_{n 3} e_{n 3}
$$

Where: $\mathrm{F}_{\mathrm{nN}}$ are components of force
$\boldsymbol{e}_{\mathrm{nN}}$ are directional vectors (unit vectors)
N is a component and direction identifier ( $\mathrm{N}=1,2,3$ )
The vectors have magnitude:

$$
\left|F_{n}\right|=F_{n 4}
$$

The magnitudes are related to components:

$$
\mathrm{F}_{\mathrm{n} 1}^{2}+\mathrm{F}_{\mathrm{n} 2}{ }^{2}+\mathrm{F}_{\mathrm{n} 3}{ }^{2}=\mathrm{F}_{\mathrm{n} 4}{ }^{2}
$$

Sub-components ( $\mathrm{F}_{\mathrm{n} 5}$ ) are also related to components: $\quad \mathrm{F}_{\mathrm{n} 5}{ }^{2}=\mathrm{F}_{\mathrm{n} 4}{ }^{2}-\mathrm{F}_{\mathrm{n} 3}{ }^{2}=\mathrm{F}_{\mathrm{n} 1}{ }^{2}+\mathrm{F}_{\mathrm{n} 2}{ }^{2}$

## Vector Geometry;

The Cartesian co-ordinates of force are: $\mathrm{F}_{\mathrm{n} 1}, \mathrm{~F}_{\mathrm{n} 2}, \mathrm{~F}_{\mathrm{n} 3}$
The Polar co-ordinates of force are: $\quad F_{n 4}, \theta_{n}, \phi_{n}$
Compliment angle $\left(\theta^{\prime}{ }_{n}\right)$ is: $\quad \theta_{n}^{\prime}=1 / 2 \pi-\theta_{n}$
It is convenient to write: $\quad A_{n}=\phi_{n}$

$$
B_{n}=\theta_{n}^{\prime}
$$

A 3D vector has angular geometry:

| $\mathrm{F}_{\mathrm{n} 1}=\mathrm{F}_{\mathrm{n} 5} \operatorname{Cos}\left(\mathrm{~A}_{\mathrm{n}}\right)$ | and; | $\mathrm{F}_{\mathrm{n} 2}=\mathrm{F}_{\mathrm{n} 5} \operatorname{Sin}\left(\mathrm{~A}_{\mathrm{n}}\right)$ |
| :--- | :--- | :--- |
| $\mathrm{F}_{\mathrm{n} 5}=\mathrm{F}_{\mathrm{n} 4} \operatorname{Cos}\left(\mathrm{~B}_{\mathrm{n}}\right)$ | and; | $\mathrm{F}_{\mathrm{n} 3}=\mathrm{F}_{\mathrm{n} 4} \operatorname{Sin}\left(\mathrm{~B}_{\mathrm{n}}\right)$ |

## Reciprocation Rules;

Reciprocating forces always act upon the same particle. Two force vectors ( $\boldsymbol{F}_{\mathbf{1}}, \boldsymbol{F}_{\mathbf{8}}$ ) are reciprocal, acting upon $P_{1}$. The rules for reciprocal vectors are:

Rule 11: $\quad\left|F_{1}\right|=\left|F_{8}\right| \quad$ giving: $F_{14}=F_{84} \quad$ (reciprocal magnitudes)
Rule 12: $\quad \mathrm{A}_{1}=\mathrm{A}_{8} \quad$ (reciprocal angles)
Rule 13: $\quad \mathrm{B}_{1}=\mathrm{B}_{8} \quad$ (reciprocal angles)
Rule 14: $\quad F_{11}+F_{81}=0 \quad$ (reciprocal components)

Reciprocal geometry gives:

$$
\begin{array}{lllll}
\mathrm{F}_{14}-\mathrm{F}_{84}=0 & \text { and: } & \mathrm{A}_{1}-\mathrm{A}_{8}=0 & \text { and: } & \mathrm{B}_{1}-\mathrm{B}_{8}=0 \\
\mathrm{~F}_{11}+\mathrm{F}_{81}=0 & \text { and: } & \mathrm{F}_{12}+\mathrm{F}_{82}=0 & \text { and: } & \mathrm{F}_{13}+\mathrm{F}_{83}=0
\end{array}
$$

## Special Reciprocation;

The exchange force $\left(F_{1}\right)$ and the primary force $\left(F_{8}\right)$ also have "special reciprocation".

The rules of special reciprocation are:
Rule 21: $\quad A_{1}=B_{1} \quad$ (also giving; $A_{1}=B_{1}=A_{8}=B_{8}$ ) (see Rules 12, 13)

$$
\operatorname{Cos}\left(\mathrm{A}_{1}\right)=\operatorname{Cos}\left(\mathrm{B}_{1}\right)
$$

From vector geometry: $F_{11} / F_{15}=F_{15} / F_{14}$
Rule 22: $\quad F_{11} F_{14}=F_{15}{ }^{2}$

Also: Rule 23: $\quad F_{11} F_{14}=F_{15}^{2}$

Rule 14 gives: $\quad F_{11}+F_{81}=0$

$$
\left.F_{15}^{2} / F_{14}+F_{85}^{2} / F_{84}=0 \quad \text { (see Rules } 22,23\right)
$$

$\mathrm{F}_{15}{ }^{2}+\mathrm{F}_{85}{ }^{2}=0 \quad$ (see Rule 11)

Rule 24: $\quad \mathrm{F}_{15}=\mathrm{iF}_{85} \quad$ where $: \mathrm{i}^{2}+1=0$

The complex constant (i) is considered to represent a ratio of special reciprocity: $i=F_{15} / F_{85}$

## Component Rules;

Components of the ten vectors are related by "component rules of structure". These rules help to define the interactive structure. Congruent components are spatially co-incident. Three sets of rules are required: angular rules, congruency rules, and non-congruency rules.

The "angular rules" relate angles associated with different vectors:
Rule 31: $\quad \mathrm{A}_{1}=\mathrm{A}_{7}$
Rule 32: $\quad B_{1}=B_{7}$

Rule 33: $\quad \mathrm{A}_{9}=\mathrm{B}_{9}$
The "congruency rules" identify "equal and spatially congruent" components. The congruency rules are:
Rule 41: $\quad \mathrm{F}_{14}=\mathrm{F}_{64}$
(exchange pair) (opposite directions)
Rule 42: $\quad \mathrm{F}_{24}=\mathrm{F}_{54}$
(exchange pair) (opposite directions)
Rule 43: $\quad \mathrm{F}_{34}=\mathrm{F}_{44}$
(exchange pair) (opposite directions)
The "non-congruency rules" relate components that are equal, but not congruent and not reciprocal:
Rule 51: $\quad F_{54}=F_{15}+F_{94}$
Rule 52: $\quad \mathrm{F}_{95}=\mathrm{F}_{04}$
Rule 53: $\quad \mathrm{F}_{94}=\mathrm{F}_{95}{ }^{2} / \mathrm{F}_{91}$
Rule 54: $\quad \mathrm{F}_{63}=\mathrm{F}_{15}$
Rule 55: $\quad \mathrm{F}_{65}=2 \mathrm{~F}_{95}$
Rule 56: $\quad \mathrm{F}_{54}=\mathrm{F}_{75}$

## The Force Equations;

The component rules lead to three scalar force equations, which relate components of force from different vectors. The three "force equations" are:

The Schrodinger force equation
The Klein-Gordon force equation
The Dirac force equation
The Schrodinger force equation is:
$\mathrm{F}_{15}=\mathrm{F}_{54}-\mathrm{F}_{94}$
(see rule 51) (primary force rule)
$\mathrm{iF}_{85}=\mathrm{F}_{54}-\mathrm{F}_{94}$
(see rule 24)
$\mathrm{iF}_{85}=\mathrm{F}_{54}-\mathrm{F}_{95}{ }^{2} / \mathrm{F}_{91}$
(see rule 53)

The Klein-Gordon force equation is:

$$
\begin{array}{ll}
F_{63}{ }^{2}=F_{64}{ }^{2}-F_{65}{ }^{2} & \text { (vector geometry) } \\
F_{15}{ }^{2}=F_{64}{ }^{2}-F_{65}{ }^{2} & \text { (see rule 54) } \\
-F_{85}{ }^{2}=F_{64}{ }^{2}-F_{65}{ }^{2} & \text { (see rule 24) } \\
-F_{85}{ }^{2}=F_{64}{ }^{2}-4 F_{95}{ }^{2} & \text { (see rule 55) }
\end{array}
$$

The Dirac force equation is:

| $\mathrm{F}_{15}=\mathrm{F}_{54}-\mathrm{F}_{94}$ | (see rule 51) (primary force rule) |
| :--- | :--- |
| $\mathrm{iF}_{85}=\mathrm{F}_{54}-\mathrm{F}_{94}$ | (see rule 24) |
| $\mathrm{iF}_{85}=\mathrm{F}_{75}-\mathrm{F}_{94}$ | (see rule 56) |
| $\mathrm{iF}_{85}=\mathrm{F}_{74} \operatorname{Cos}\left(\mathrm{~B}_{7}\right)-\mathrm{F}_{94}$ | (from vector geometry) |
| $\mathrm{iF}_{85}=\mathrm{F}_{74} \operatorname{Cos}\left(\mathrm{~B}_{7}\right)-\mathrm{F}_{95}{ }^{2} / \mathrm{F}_{91}$ | (see rule 53) |
| $\mathrm{iF}_{85}=\mathrm{F}_{74} \operatorname{Cos}\left(\mathrm{~B}_{7}\right)-\mathrm{F}_{04}{ }^{2} / \mathrm{F}_{91}$ | (see rule 52) |
| $\mathrm{iF}_{85}=\mathrm{F}_{74} \operatorname{Cos}\left(\mathrm{~B}_{7}\right)-\left(\mathrm{F}_{01}{ }^{2}+\mathrm{F}_{02}{ }^{2}+\mathrm{F}_{03}{ }^{2}\right) / \mathrm{F}_{91}$ | (from vector geometry) |

In summary the three scalar force equations are:

$$
\begin{array}{ll}
\mathrm{iF}_{85}=\mathrm{F}_{54}-\mathrm{F}_{95}{ }^{2} / \mathrm{F}_{91} & \text { ("Schrodinger Force" equation) } \\
-\mathrm{F}_{85}{ }^{2}=\mathrm{F}_{64}{ }^{2}-4 \mathrm{~F}_{95}{ }^{2} & \text { ("Klein-Gordon Force" equation) } \\
\mathrm{iF}_{85}=\mathrm{F}_{74} \operatorname{Cos}\left(\mathrm{~B}_{7}\right)-\left(\mathrm{F}_{01}{ }^{2}+\mathrm{F}_{02}{ }^{2}+\mathrm{F}_{03}{ }^{2}\right) / \mathrm{F}_{91} & \text { ("Dirac Force" equation) }
\end{array}
$$

## The Energy Equations;

The force equations transform to energy equations. The transformation rule is: $E_{n N}=\lambda_{P} F_{n N}$ Where: $\lambda_{P}$ is Plank wavelength: $\lambda_{P}=\left(2 \pi G h / c^{3}\right)^{1 / 2}$
n is a vector identifier $\quad(\mathrm{n}=0 . . .9)$
N is a component and sub-component identifier ( $\mathrm{N}=1 \ldots . .5$ )

Transformation gives three energy equations:
The "Schrodinger Energy Equation" is: $\mathrm{iE}_{85}=\mathrm{E}_{54}-\mathrm{E}_{95}{ }^{2} / \mathrm{E}_{91}$
The "Klein-Gordon Energy Equation" is: $-\mathrm{E}_{85}{ }^{2}=\mathrm{E}_{64}{ }^{2}-4 \mathrm{E}_{95}{ }^{2}$
The "Dirac Energy Equation" is: $\quad \mathrm{i}_{85}=\mathrm{E}_{74} \operatorname{Cos}\left(\mathrm{~B}_{7}\right)-\left(\mathrm{E}_{01}{ }^{2}+\mathrm{E}_{02}{ }^{2}+\mathrm{E}_{03}{ }^{2}\right) / \mathrm{E}_{91}$

## Operators;

A temporal operator is:
$\partial / \partial t$
The second order temporal operator is: $\partial^{2} / \partial t^{2}$
A spatial operator is:
$\partial / \partial R$

The second order spatial operator is the La Place operator: $\partial^{2} / \partial R^{2}=\partial^{2} / \partial x^{2}+\partial^{2} / \partial y^{2}+\partial^{2} / \partial z^{2}$ Where: $x, y, z$ are the Cartesian co-ordinates of space

## Frequencies;

Two frequencies $\left(f_{T}, f_{R}\right)$ are required, each frequency is associated with an operator:

$$
\begin{array}{lll}
f_{T}=\partial / \partial \mathrm{t} & \text { and: } & f_{T}{ }^{2}=\partial^{2} / \partial \mathrm{t}^{2} \\
f_{R}=\partial \mathrm{v} / \partial \mathrm{R} & \text { and: } & f_{R}{ }^{2}=\partial^{2} \mathrm{v}^{2} / \partial \mathrm{R}^{2}=\left(\partial^{2} / \partial \mathrm{R}^{2}\right) \mathrm{v}^{2}
\end{array}
$$

Where: v is velocity

## Energy Definitions;

Energies may be defined as follows.

$$
\begin{array}{lll}
\mathrm{E}_{54}=(\mathrm{v} / \mathrm{c}) \mathrm{V}_{\mathrm{y}} & \mathrm{E}_{64}=(\mathrm{v} / \mathrm{c}) \mathrm{m}_{3} \mathrm{c}^{2} & \mathrm{E}_{74}=(\mathrm{v} / \mathrm{c}) \mathrm{m}_{1} \mathrm{c}^{2} \\
\mathrm{E}_{85}=(\mathrm{v} / \mathrm{c}) \mathrm{B} f_{T} & \mathrm{E}_{91}=1 / 2 \mathrm{~m}_{2} \mathrm{vc} & \mathrm{E}_{95}=1 / 2 \mathrm{~F} f_{R} \\
\mathrm{E}_{01}{ }^{2}=\mathrm{E}_{012} \mathrm{E}_{91} & \mathrm{E}_{02}{ }^{2}=\mathrm{E}_{022} \mathrm{E}_{91} & \mathrm{E}_{03}{ }^{2}=\mathrm{E}_{03 \mathrm{a}} \mathrm{E}_{91}
\end{array}
$$

Where: EONa is Dirac energy: $\quad E_{O N a}=a_{O N} p_{o n v}$
$a_{0 N}$ is a ratio
$p_{o n}$ is Dirac momentum
$v$ is velocity
万 is the reduced Plank constant
c is the light constant
$f_{T}, f_{R}$ are frequencies
$\mathrm{m}_{\mathrm{x}}$ is dynamic mass（ x is a particle identifier）
$\mathrm{V}_{\mathrm{y}}$ is potential

## Defined Schrodinger Energy；

The energy equations may include defined energies．
The＂Schrodinger Energy equation＂is：$\quad \mathrm{iE}_{85}=\mathrm{E}_{54}-\mathrm{E}_{95}{ }^{2} / \mathrm{E}_{91}$
Defined energies give：
$\mathrm{i}(\mathrm{v} / \mathrm{c}) \mathrm{b} f_{T}=(\mathrm{v} / \mathrm{c}) \mathrm{V}_{\mathrm{y}}-\left(1 / 2 Ђ f_{R}\right)^{2} /\left(1 / 2 \mathrm{~m}_{2} \mathrm{vc}\right)$
$\mathrm{i}(\mathrm{v} / \mathrm{c}) \mathrm{b} f_{T}=(\mathrm{v} / \mathrm{c}) \mathrm{V}_{\mathrm{y}}-\left(\mathrm{万} f_{R}\right)^{2} /\left(2 \mathrm{~m}_{2} \mathrm{vc}\right)$
$\mathrm{i}(\mathrm{v} / \mathrm{c}) \mathrm{b} f_{T}=(\mathrm{v} / \mathrm{c}) \mathrm{V}_{\mathrm{y}}-(\mathrm{v} / \mathrm{c})\left(万 f_{R}\right)^{2} /\left(2 \mathrm{~m}_{2} \mathrm{v}^{2}\right)$
$\mathrm{i}(\mathrm{v} / \mathrm{c}) \mathrm{b}_{T}=(\mathrm{v} / \mathrm{c}) \mathrm{V}_{\mathrm{v}}-(\mathrm{v} / \mathrm{c})\left(万^{2} / 2 \mathrm{~m}_{2}\right)\left(f_{R}^{2} / \mathrm{v}^{2}\right)$
giving＂Defined Schrodinger Energy＂：$\quad i(v / c) \zeta \partial / \partial t=(v / c) V_{y}-(v / c)\left(万^{2} / 2 m_{2}\right) \partial^{2} / \partial R^{2}$
Where： $\mathrm{v} / \mathrm{c}$ is the relativistic ratio
Non－relativistic Schrodinger Energy is：$\quad i 万 \partial / \partial t=V_{y}-\left(万^{2} / 2 m_{2}\right) \partial^{2} / \partial R^{2}$
A wavefunction $(\psi)$ interacts with energy，giving the Schrodinger equation：

$$
i Ђ(\partial \psi / \partial t)=V_{y} \psi-\left(万^{2} / 2 m_{2}\right)\left(\partial^{2} \psi / \partial R^{2}\right)
$$

## Defined Klein－Gordon Energy；

The＂Klein－Gordon Energy Equation＂is：$-\mathrm{E}_{85}{ }^{2}=\mathrm{E}_{64}{ }^{2}-4 \mathrm{E}_{95}{ }^{2}$

Defined energies give：

$$
\begin{aligned}
& 0=E_{64}{ }^{2}-4 \mathrm{E}_{95}{ }^{2}+\mathrm{E}_{85}{ }^{2} \\
& 0=(\mathrm{v} / \mathrm{c})^{2} m_{3}^{2} c^{4}-4\left(1 / 2 \text { 万f } f_{R}\right)^{2}+(\mathrm{v} / \mathrm{c})^{2}\left(万 f_{T}\right)^{2} \\
& 0=m_{3}^{2} c^{2}-万^{2}\left(f_{R}^{2} / v^{2}\right)+(1 / c)^{2}\left(万 f_{T}\right)^{2} \\
& 0=m_{3}^{2} c^{2} / 万^{2}-\left(f_{R}^{2} / v^{2}\right)+(1 / c)^{2} f_{T}^{2}
\end{aligned}
$$

$$
\text { giving "Defined K-G Energy": } \quad 0=m_{3}^{2} c^{2} / 丂^{2}-\partial^{2} / \partial R^{2}+\left(1 / c^{2}\right) \partial^{2} / \partial t^{2}
$$

A wave function interacts with energy giving the Klein－Gordon Equation：

$$
0=\left(m_{3}{ }^{2} c^{2} / Ђ^{2}\right) \psi-\partial^{2} \psi / \partial R^{2}+\left(1 / c^{2}\right) \partial^{2} \psi / \partial t^{2}
$$

## Defined Dirac Energy；

The＂Dirac Energy Equation＂is： $\mathrm{E}_{85}=\mathrm{E}_{74} \operatorname{Cos}\left(\mathrm{~B}_{7}\right)-\left(\mathrm{E}_{01}{ }^{2}+\mathrm{E}_{02}{ }^{2}+\mathrm{E}_{03}{ }^{2}\right) / \mathrm{E}_{91}$

$$
\begin{aligned}
& \mathrm{i} \mathrm{E}_{85}=\mathrm{E}_{74} \mathrm{Cos}\left(\mathrm{~B}_{7}\right)-\left(\mathrm{E}_{01 \mathrm{a}} \mathrm{E}_{91}+\mathrm{E}_{02 \mathrm{a}} \mathrm{E}_{91}+\mathrm{E}_{03 \mathrm{a}} \mathrm{E}_{91}\right) / \mathrm{E}_{91} \\
& \mathrm{E}_{85}=\mathrm{E}_{74} \mathrm{Cos}\left(\mathrm{~B}_{7}\right)-\left(\mathrm{E}_{01 \mathrm{a}}+\mathrm{E}_{02 \mathrm{a}}+\mathrm{E}_{03 \mathrm{a}}\right)
\end{aligned}
$$

Defined energies give：$\quad i(v / c) B f_{T}=(v / c) m_{1} c^{2} \operatorname{Cos}\left(B_{7}\right)-\left(a_{01} p_{011} v+a_{02} p_{02} v+a_{03} p_{03} v\right)$

$$
\mathrm{i}(\mathrm{v} / \mathrm{c}) \mathrm{b} f_{T}=(\mathrm{v} / \mathrm{c}) \mathrm{m}_{1} \mathrm{c}^{2} \operatorname{Cos}\left(\mathrm{~B}_{7}\right)-(\mathrm{v} / \mathrm{c})\left(\mathrm{a}_{01} \mathrm{p}_{01} \mathrm{c}+\mathrm{a}_{02} \mathrm{p}_{02} \mathrm{c}+\mathrm{a}_{03} \mathrm{p}_{03} \mathrm{c}\right)
$$

$$
i 万 f_{T}=m_{1} c^{2} \operatorname{Cos}\left(B_{7}\right)-c\left(a_{01} p_{01}+a_{02} p_{02}+a_{03} p_{03}\right)
$$

Defined Dirac Energy：$\quad i 万 \partial / \partial t=\left(m_{1} c^{2}\right) \operatorname{Cos}\left(B_{7}\right)-c\left(a_{01} p_{01}+a_{02} p_{02}+a_{03} p_{03}\right)$
Dirac energy interacts with a waveform（ $\psi$ ）．
Giving the Dirac Equation：$\quad i 万 \partial \psi / \partial t=\psi\left(m_{1} c^{2}\right) \operatorname{Cos}\left(B_{7}\right)-c \psi\left(a_{01} p_{01}+a_{02} p_{02}+a_{03} \mathrm{p}_{03}\right)$

## Dirac Momentum；

Dirac momentum is represented as a vector $\left(\boldsymbol{p}_{0}\right): \quad \boldsymbol{p}_{0}=\mathrm{p}_{01} \boldsymbol{e}_{01}+\mathrm{p}_{02} \boldsymbol{e}_{02}+\mathrm{p}_{03} \boldsymbol{e}_{03}$
Vector magnitude：$\quad\left|\boldsymbol{p}_{0}\right|=p_{04}$
Components：$\quad \mathrm{p}_{01}{ }^{2}+\mathrm{p}_{02}{ }^{2}+\mathrm{p}_{03}{ }^{2}=\mathrm{p}_{04}{ }^{2}$
Sub－component：$\quad \mathrm{p}_{05}{ }^{2}=\mathrm{p}_{01}{ }^{2}+\mathrm{p}_{02}{ }^{2}=\mathrm{p}_{04}{ }^{2}-\mathrm{p}_{03}{ }^{2}$
Momentum definitions：

$$
\begin{array}{llll}
\mathrm{p}_{01}=\mathrm{h} / \lambda_{0} & \mathrm{p}_{02}=(\mathrm{v} / \mathrm{c})(\mathrm{h} / \lambda) & \mathrm{p}_{03}=\mathrm{m}_{0} \mathrm{c} & \mathrm{p}_{04}=\mathrm{mc} \\
\mathrm{p}_{05}=\mathrm{mv}=\mathrm{h} / \lambda & \text { (De Broglie momentum })
\end{array}
$$

Vector geometry gives：

$$
\begin{array}{ll}
\operatorname{Cos}\left(A_{0}\right)=p_{01} / p_{05}=\lambda / \lambda_{0} & \operatorname{Sin}\left(A_{0}\right)=p_{02} / p_{05}=v / c \\
\operatorname{Cos}\left(B_{0}\right)=p_{05} / p_{04}=v / c & \operatorname{Sin}\left(B_{0}\right)=p_{03} / p_{04}=m_{0} / m
\end{array}
$$

The relativistic ratios $\left(\mathrm{v} / \mathrm{c}, \mathrm{m}_{0} / \mathrm{m}, \lambda / \lambda_{0}\right)$ give dilation and contraction：

$$
\begin{array}{lll}
\cos ^{2}\left(A_{0}\right)+\operatorname{Sin}^{2}\left(A_{0}\right)=1 & \text { giving wavelength contraction: } & \left(\lambda / \lambda_{0}\right)^{2}+(\mathrm{v} / \mathrm{c})^{2}=1 \\
\operatorname{Cos}^{2}\left(B_{0}\right)+\operatorname{Sin}^{2}\left(B_{0}\right)=1 & \text { giving mass dilation: } & (\mathrm{v} / \mathrm{c})^{2}+\left(\mathrm{m}_{0} / \mathrm{m}\right)^{2}=1
\end{array}
$$

Dirac "sub-energies" (EONa) are: $\mathrm{E}_{\text {ONa }}=\mathrm{a}_{\mathrm{ON}} \mathrm{p}_{\mathrm{ON}} \mathrm{V}$

$$
\begin{aligned}
& \mathrm{E}_{01 \mathrm{a}}=\mathrm{a}_{01} \mathrm{p}_{011} \mathrm{v}=\mathrm{a}_{01}\left(\mathrm{~h} / \lambda_{0}\right) \mathrm{v}=\mathrm{a}_{01} \mathrm{hv} / \lambda_{0} \\
& \mathrm{E}_{02 \mathrm{a}}=\mathrm{a}_{02} \mathrm{p}_{02} \mathrm{v}=\mathrm{a}_{02}(\mathrm{vh} / \mathrm{c} \lambda) \mathrm{v}=\mathrm{a}_{02}(\mathrm{v} / \mathrm{c})(\mathrm{hv} / \lambda)=\mathrm{a}_{02} \mathrm{~h}_{\mathrm{x}} \mathrm{v} / \lambda \quad\left(\text { where } ; \mathrm{h}_{\mathrm{x}}=\mathrm{vh} / \mathrm{c}\right) \\
& \mathrm{E}_{03 \mathrm{a}}=\mathrm{a}_{03} \mathrm{p}_{03} \mathrm{v}=\mathrm{a}_{03}\left(\mathrm{~m}_{0} \mathrm{c}\right) \mathrm{v}=\mathrm{a}_{03} \mathrm{~m}_{0} \mathrm{vc} \\
& \mathrm{E}_{04 \mathrm{a}}=\mathrm{a}_{04} \mathrm{p}_{04} \mathrm{v}=\mathrm{a}_{04}(\mathrm{mc}) \mathrm{v}=\mathrm{a}_{04} \mathrm{mvc} \\
& \mathrm{E}_{05 \mathrm{a}}=\mathrm{a}_{05} \mathrm{p}_{05} \mathrm{v}=\mathrm{a}_{05}(\mathrm{~h} / \lambda) \mathrm{V}=\mathrm{a}_{05} \mathrm{hv} / \lambda=\mathrm{a}_{05}(\mathrm{mv}) \mathrm{v}=\mathrm{a}_{05} \mathrm{mv}^{2}
\end{aligned}
$$

## The Compton Equation;

The Compton equation represents an interaction between a photon $\left(P_{2}\right)$ and an electron $\left(P_{1}\right)$.
The equation may be written as: $\quad \lambda_{3}=4 \pi \hbar \operatorname{Sin}^{2}\left(1 / 2 \theta_{c}\right) / m_{1} c$

$$
\lambda_{3}=h \operatorname{Sin}^{2}\left(1 / 2 \theta_{\mathrm{c}}\right) /\left(1 / 2 \mathrm{~m}_{1} \mathrm{c}\right)
$$

Where: $\lambda_{3}$ is the wavelength of the mediating particle $\left(P_{3}\right): \lambda_{3}=\lambda_{2 b}-\lambda_{2 a}$
$\lambda_{2 a}$ is the initial wavelength of the photon
$\lambda_{2 b}$ is the final wavelength of the photon
$\theta_{c}$ is the scattering angle of the photon
$m_{1}$ is the rest mass of the electron
The interaction may be represented as forces.
The exchange force $\left(F_{6}\right)$ acts between the mediating particle (P3) and the electron (P1) having magnitude:

$$
\left|F_{6}\right|=F_{64}=h c / r \lambda_{3} \quad\left(\text { where } ; r \text { is the distance between } P_{3} \text { and } P_{1}\right)
$$

The exchange force $\left(F_{1}\right)$ acts in the opposite direction, between the electron ( P 1 ) and the mediating particle (P3) having magnitude:

$$
\left|\boldsymbol{F}_{1}\right|=\mathrm{F}_{14}=\mathrm{F}_{64}=\mathrm{hc} / \mathrm{r} \lambda_{3}
$$

Force component ( $F_{11}$ ) is associated with the rest mass of the electron: $F_{11}=1 / 2 m_{1} c^{2} / r$
From vector geometry: $\mathrm{F}_{11}=\mathrm{F}_{15} \operatorname{Cos}\left(\mathrm{~A}_{1}\right)$

$$
\mathrm{F}_{11}{ }^{2}=\mathrm{F}_{15}{ }^{2} \operatorname{Cos}^{2}\left(\mathrm{~A}_{1}\right)
$$

From rule 22:

$$
F_{11} F_{14}=F_{15}^{2}
$$

$$
\text { (from: } \mathrm{A}_{1}=\mathrm{B}_{1} \text { ) }
$$

Giving:

$$
\begin{aligned}
& \mathrm{F}_{11}{ }^{2}=\mathrm{F}_{11} \mathrm{~F}_{14} \operatorname{Cos}^{2}\left(\mathrm{~A}_{1}\right) \\
& \mathrm{F}_{11}=\mathrm{F}_{14} \operatorname{Cos}^{2}\left(\mathrm{~A}_{1}\right) \\
& 1 / 2 \mathrm{~m}_{1} \mathrm{c}^{2} / \mathrm{r}=\left(\mathrm{hc} / \mathrm{r}_{3}\right) \operatorname{Cos}^{2}\left(\mathrm{~A}_{1}\right) \\
& \lambda_{3}\left(1 / 2 \mathrm{~m}_{1} \mathrm{c}\right)=\mathrm{h} \cos ^{2}\left(\mathrm{~A}_{1}\right)
\end{aligned}
$$

Compliment angles: $\quad \mathrm{A}_{1}+1 / 2 \theta_{\mathrm{C}}=1 / 2 \pi$
giving:

$$
\operatorname{Cos}\left(A_{1}\right)=\operatorname{Sin}\left(1 / 2 \theta_{C}\right)
$$

Equivalent Compton equation: $\lambda_{3}=\left(2 h / m_{1} c\right) \operatorname{Sin}^{2}\left(1 / 2 \theta_{c}\right)$

## Gravitational Approximation;

Two objects ( $\mathrm{P}_{1}, \mathrm{P} 2$ ) interact gravitationally. Each object has a gravitational field. Each field is associated with "field energy" ( $\mathrm{E}_{\mathrm{G}_{1}}, \mathrm{E}_{\mathrm{G}_{2}}$ ).

Primary force $\left(\boldsymbol{F}_{7}\right)$ is associated with the combined field energies.
Force component ( $\mathrm{F}_{75}$ ) is related to field energy ( $\mathrm{E}_{61}$ ):

$$
\mathrm{F}_{75}=\mathrm{E}_{61} / \lambda_{3}
$$

Force component ( $\mathrm{F}_{73}$ ) is related to field energy ( $\mathrm{E}_{62}$ ):

$$
\mathrm{F}_{73}=\mathrm{E}_{62} / \lambda_{3}
$$

Where: $\lambda_{3}$ is the wavelength of the mediating particle $\left(P_{3}\right)$
The magnitude of "combined field force" $\left(\mathrm{F}_{74}\right)$ is:
$\mathrm{F}_{74}{ }^{2}=\mathrm{F}_{73}{ }^{2}+\mathrm{F}_{75}{ }^{2}$
Force component ( $\mathrm{F}_{72}$ ) is related to the energy ( $\mathrm{E}_{\mathrm{P} 2}$ ) of $\mathrm{P}_{2}$ : $\quad \mathrm{F}_{72}=\mathrm{E}_{\mathrm{P} 2} / \lambda_{3}$
The "gravitational rule" is: $\quad A_{7}=B_{7}$

$$
\operatorname{Sin}\left(A_{7}\right)=\operatorname{Sin}\left(B_{7}\right)
$$

Vector geometry gives: $\quad \mathrm{F}_{72} / \mathrm{F}_{75}=\mathrm{F}_{73} / \mathrm{F}_{74}$

$$
\mathrm{F}_{72} \mathrm{~F}_{74}=\mathrm{F}_{73} \mathrm{~F}_{75}
$$

The gravitational equation is: $\quad\left(\mathrm{E}_{\mathrm{P} 2} / \lambda_{3}\right) \mathrm{F}_{74}=\left(\mathrm{E}_{\mathrm{G}_{2}} / \lambda_{3}\right)\left(\mathrm{E}_{61} / \lambda_{3}\right)$
Field and particle energies may be defined.
Particle energy $\left(E_{P_{2}}\right)$ is: $\quad E_{p 2}=m_{2} v c$
De Broglie momentum gives: $\quad \mathrm{E}_{\mathrm{P} 2}=\left(\mathrm{m}_{2} \mathrm{v}\right) \mathrm{c}=\left(\mathrm{h} / \lambda_{3}\right) \mathrm{c}=\mathrm{hc} / \lambda_{3}$
Field energy $\left(\mathrm{E}_{\mathrm{G}_{1}}\right)$ is: $\quad \mathrm{E}_{\mathrm{G} 1}=(\mathrm{hcG})^{1 / 2} \mathrm{~m}_{1} / \mathrm{r}$
Field energy $\left(\mathrm{E}_{\mathrm{G} 2}\right)$ is: $\quad \mathrm{E}_{\mathrm{G} 2}=(\mathrm{hcG})^{1 / 2} \mathrm{~m}_{2} / \mathrm{r}$

Where: $r$ is the distance between the centers of $P_{1}$ and $P_{2}$

The gravitational equation is:

$$
\left(\mathrm{E}_{\mathrm{P} 2} / \lambda_{3}\right) \mathrm{F}_{74}=\left(\mathrm{E}_{\mathrm{G} 2} / \lambda_{3}\right)\left(\mathrm{E}_{\mathrm{G} 1} / \lambda_{3}\right)
$$

Giving:

$$
\left(\mathrm{hc} / \lambda_{3}^{2}\right) \mathrm{F}_{74}=\left[(\mathrm{hcG})^{1 / 2} \mathrm{~m}_{2} / \mathrm{r} \lambda_{3}\right]\left[(\mathrm{hcG})^{1 / 2} \mathrm{~m}_{1} / \mathrm{r} \lambda_{3}\right]
$$

$$
\mathrm{hcF}_{74}=(\mathrm{hcG}) \mathrm{m}_{1} \mathrm{~m}_{2} / \mathrm{r}^{2}
$$

Giving the Newton approximation: $\quad \mathrm{F}_{74}=\mathrm{Gm}_{1} \mathrm{~m}_{2} / \mathrm{r}^{2}$

The Coulomb approximation is obtained in a similar manner.

## Quantization;

Forces $\left(F_{1}, F_{8}\right)$ have special reciprocation.
From rule 12:

$$
\begin{aligned}
& A_{1}=A_{8} \\
& \operatorname{Cos}\left(A_{1}\right)=\operatorname{Cos}\left(A_{8}\right)
\end{aligned}
$$

From vector geometry: $\quad \mathrm{F}_{11} / \mathrm{F}_{15}=\mathrm{F}_{81} / \mathrm{F}_{85}$
$F_{11} F_{85}=F_{15} F_{81}$
$F_{11}{ }^{2} F_{85}{ }^{2}=F_{15}{ }^{2} F_{81}{ }^{2}$
Rule 23 gives:

$$
F_{11}^{2} F_{85}^{2}=\left(F_{11} F_{14}\right) F_{81}^{2}
$$

$$
\mathrm{F}_{11} \mathrm{~F}_{85}{ }^{2}=\mathrm{F}_{14} \mathrm{~F}_{81}{ }^{2}
$$

Rules 24 and 41 give:

$$
F_{11}\left(-F_{15}^{2}\right)=F_{64} F_{81}^{2}
$$

Associated energies are:
$E_{n N}=\lambda F_{n N}$

Giving energies:

$$
-\mathrm{E}_{11} \mathrm{E}_{15}^{2}=\mathrm{E}_{64} \mathrm{E}_{81}{ }^{2}
$$

Energy definitions are: $E_{11}=1 / 2 m_{e} c^{2}$
$\mathrm{E}_{15}=\mathrm{e}^{2} / 4 \pi \varepsilon_{0} r$ $\mathrm{E}_{81}=\mathrm{hc} / \lambda$

Where: $r$ is the distance between the centers of $P_{1}$ and $P_{3}$
Substitution gives:

$$
\begin{aligned}
& -\left(1 / 2 m_{e} c^{2}\right)\left(e^{2} / 4 \pi \varepsilon_{0} r\right)^{2}=E_{64}(h c / \lambda)^{2} \\
& E_{64}=-\left(m_{e} e^{4} / 8 \varepsilon_{0}^{2} h^{2}\right)\left(\lambda^{2} / 4 \pi^{2} r^{2}\right)
\end{aligned}
$$

The quantization rule states that: only an integer number ( $n$ ) of wavelength $(\lambda)$ may be imposed upon the circumference ( $C_{r}$ ) of a circle.
giving quantum energy:

$$
\mathrm{n} \lambda=\mathrm{C}_{\mathrm{r}}=2 \pi \mathrm{r}
$$

$$
E_{64}=-\left(m_{e} e^{4} / 8 \varepsilon_{0}^{2} h^{2}\right)\left(1 / n^{2}\right)
$$

## Uncertainty；

Experiment includes multiple observations of position $\left(R_{i}\right)$ and momentum（ $P_{i}$ ）．

Where：＇ i ＇is an observation identifier（or experiment number）
＂Positional deviation＂$\left(r_{i}\right)$ is：$r_{i}=R_{i}-R_{M}$
Where：$R_{M}$ is the mean value of all observed positions
The combined positional deviation is represented as a vector $\left(\boldsymbol{V}_{r}\right)$ having components $\left(r_{i}\right)$ ．

The vector has magnitude representing total deviation：$\left|V_{r}\right|=r_{T}$
The magnitude is related to components：$\quad r_{T}{ }^{2}=\sum r_{i}{ }^{2}\{i=1, N\}$
Where： N is the total number of observations
The standard deviation for position $\left(\sigma_{r}\right)$ is：
$\sigma_{r}=r_{T} /(N-1)^{1 / 2}$
The standard deviation for momentum $\left(\sigma_{p}\right)$ is：
$\sigma_{\mathrm{p}}=\mathrm{p}_{\mathrm{T}} /(\mathrm{N}-1)^{1 / 2}$
The uncertainty $\left(U_{r p}\right)$ is：
$U_{r p}=\sigma_{r} \sigma_{p} \geq 1 / 2$ 万 $U_{\mathrm{rp}}=r_{\mathrm{T}} \mathrm{p}_{\mathrm{T}} /(\mathrm{N}-1) \geq 1 / 2$ 万

Where：万 is the reduced Plank constant

The uncertainty may be＂quantized＂using a quantum number（n）．
Where： n is an integer $(\mathrm{n} \geq 1)$
giving the＂quantum uncertainty equation＂：
$r_{T} p_{T} /(N-1)=1 / 2 n 万$
The＂quantization rule＂is：$\quad n \lambda=C_{r}=2 \pi R$
The quantum uncertainty equation may be written as：$\quad r_{T} p_{T} /(N-1)=1 / 2(2 \pi R / \lambda)$ 万
$r_{T} p_{T} /(N-1)=1 / 2(2 \pi 万)(R / \lambda)$
$r_{T} p_{T}=1 / 2 h(R / \lambda)(N-1)$
$p_{T} c / R+1 / 2 h c / \lambda r_{T}=1 / 2 N h c / \lambda r_{T}$
giving Rule 51：
also：
$F_{15}+F_{94}=F_{54}$
$\mathrm{F}_{54}=\mathrm{NF}_{94}$
The primary force rule is the quantum uncertainty equation．

## Conclusion;

An arrangement of particles and force vectors is represented as an "Interaction Diagram". Components and geometry of the vectors are related by "rules of structure". The primary rule is a statement of uncertainty. The interaction Diagram represents the total interaction of two particles.

