25 Klein-Gordon Derivation

If suitable operators are defined, the Klein-Gordon equation may be derived from a transformation of mass dilation. The transform uses: De Broglie momentum, quantization rules, frequency definition, and suitable operators. Quantization returns the reduced Plank constant. The transformation of mass dilation gives an “equation of motion” which interacts with a wave function, giving an equivalent Klein-Gordon equation.

Mass Dilation:

The equation for mass dilation is: $1 = (m_0/m)^2 + (v/c)^2$

Where: m_0 is rest mass

m is dynamic mass

v is scalar velocity

c is the light constant

Momentum:

Mass dilation may be written as: $1 = (m_0v/mv)^2 + (v/c)^2$

The de Broglie momentum is: $mv = h/\lambda$

Where: h is the Plank constant

λ is wavelength

Giving “de Broglie dilation”: $1 = (m_0v)^2/(h/\lambda)^2 + (v/c)^2$

Energy:

The mass dilation equation may be re-written as “Energy dilation”:

$$(hc/\lambda)^2 = (m_0vc)^2 + (hv/\lambda)^2$$

Quantization:

Assume that a “wave” may be composed of a discreet number (n) of “wave packets”. The wave has a wavelength (λ) and a wave packet also has wavelength (λ_p).

Giving: $\lambda = n\lambda_p$

Where: n is an integer

Energy dilation may be written as: $h^2c^2/n^2\lambda_p^2 = m_0^2v^2c^2 + h^2v^2/n^2\lambda_p^2$

Assume that a wave may be "free" to travel (as radiation) or may be "bound" to matter. Assume that a "bound wave" is restricted to the surface of a "wave sphere". The wave sphere has a radius (r_p) and a circumference (c_p).

A "quantization rule" states that the wavelength of a bound wave must equal the circumference of the restrictive wave sphere:

$$\lambda = c_p$$

giving: $n\lambda_p = 2\pi r_p$

Energy dilation may be written as: $h^2c^2/4\pi^2r_p^2 = m_0^2v^2c^2 + h^2v^2/4\pi^2r_p^2$

giving "quantized energy dilation": $\hbar^2c^2/r_p^2 = m_0^2v^2c^2 + \hbar^2v^2/r_p^2$

Where: ħ is the reduced Plank constant or "quantized Plank constant": $\hbar = h/2\pi$

Frequency:

Assume that a bound wave travels upon the surface of a wave sphere at light speed.

Assume the frequency (f_p) is: $f_p = c/r_p$

Quantized energy dilation becomes: $\hbar^2 f_p^2 = m_0^2v^2c^2 + \hbar^2(v^2/c^2)f_p^2$

Giving frequency-velocity ratios: $f_p^2/v^2 = m_0^2c^2/\hbar^2 + f_p^2/c^2$

An "equation of motion" is: $(f_p/v)^2 = (f_0/v)^2 + (f_p/c)^2$

Where: $m_0vc = \hbar f_0$

Relativistic Quantum Energy:

Quantized energy dilation is: $\hbar^2 f_p^2 = m_0^2v^2c^2 + \hbar^2(v^2/c^2)f_p^2$

$$\hbar^2 f_p^2 = (v^2/c^2)m_0^2c^4 + \hbar^2(v^2/c^2)f_p^2$$

Giving "relativistic quantum energy": $(\hbar f_p)^2 = (M_Rc^2)^2 + (\hbar f_R)^2$

Where: M_R is relativistic mass: $M_R = (v/c)m_0$

f_R is relativistic frequency: $f_R = (v/c)f_p$

Operators:

Two operators are required: $\partial^2/\partial t^2 = f_p^2$ is a frequency operator

$\partial^2/\partial R^2$ is a spatial operator

The ratio of operators is: $(\partial^2/\partial t^2)(\partial R^2/\partial^2) = \partial R^2/\partial t^2 = v^2$

$f_p{}^2/v^2 = \partial^2/\partial R^2$

The "equation of motion" is: $(f_p/v)^2 = (f_0/v)^2 + (f_p/c)^2$

$f_p{}^2/v^2 = m_0{}^2c^2/\hbar^2 + (1/c^2)f_p{}^2$

Substituting operators gives the "operator equation of motion":

$$\partial^2/\partial R^2 = m_0{}^2c^2/\hbar^2 + (1/c^2)\partial^2/\partial t^2$$

The Wave Function;

A wave function (ψ) interacts with the operator equation of motion:

$$\partial^2\psi/\partial R^2 = m_0{}^2c^2\psi/\hbar^2 + (1/c^2)\partial^2\psi/\partial t^2$$

giving the Klein-Gordon equivalent:

$$0 = (1/c^2)\partial^2\psi/\partial t^2 - \partial^2\psi/\partial R^2 + m_0{}^2c^2\psi/\hbar^2$$

Conclusion;

If suitable operators are defined the Klein-Gordon equation may be derived from mass dilation using De Broglie momentum and quantization rules. Quantization gives the reduced Plank constant. The "equation of motion" interacts with a wave function giving an equivalent Klein-Gordon equation.