33 Reissner–Nordström Vectors

Displacement may be represented as a 4D vector. Two types of displacement are; “length scale” and “incremental distance” both are represented as vectors. The displacement vectors may be named the “Reissner-Nordström vectors” and they act upon a common point which is also the center of three spatial regions. Vector components may be related giving the Reissner-Nordström metric.

Spatial Dimensions;

The Cartesian dimensions of space are; x, y, z

The Polar dimensions of space are; r, θ, ϕ

The co-ordinates are related; $x^2 + y^2 + z^2 = r^2$ and; $x^2 + y^2 = r^2 - z^2 = w^2$

$z = r\cos(\theta)$ and; $w = r\sin(\theta)$

$x = w\cos(\phi) = r\sin(\theta)\cos(\phi)$ and; $y = w\sin(\phi) = r\sin(\theta)\sin(\phi)$

Two arc-lengths (U_θ , U_ϕ) are associated with the angles (θ, ϕ);

$\theta = U_\theta/r$ and; $\phi = U_\phi/w$

giving incremental arc lengths; $r\partial\theta = \partial U_\theta$ and; $w\partial\phi = \partial U_\phi$

Spatial Regions;

Three spatial regions are all spherical and of different size. They all have a common center and therefore have a concentric arrangement. Each region has a radius (R, r_6, r_7) and a corresponding surface area (S_R , S_{r6} , S_{r7}). The surface areas are;

$S_R = 4\pi R^2$ $S_{r6} = 4\pi r_6^2$ $S_{r7} = 4\pi r_7^2$

The surface areas are related; $S_R^2 = S_{r6}^2 + S_{r7}^2$

Where; $S_R\text{Cos}(\beta) = S_{r6}$ and; $S_R\text{Sin}(\beta) = S_{r7}$ and; $\text{Tan}(\beta) = S_{r7}/S_{r6} = r_7^2/r_6^2$

β is the angle of “regional scale”

It is useful to note the trig ratios of angle 2β;

$S_R^2\text{Cos}(2\beta) = S_{r6}^2 - S_{r7}^2$ and; $S_R^2\text{Sin}(2\beta) = 2S_{r6}S_{r7}$

Giving; $\text{Sin}(2\beta) = 2S_{r6}S_{r7}/S_R^2 = 2S_{r6}S_{r7}/(S_{r6}^2 + S_{r7}^2) = 2r_6^2r_7^2/(r_6^4 + r_7^4)$

Displacement Vectors;

Two 4D vectors of displacement share a common origin with each other and with the three spatial regions. One vector (***r***) represents "length scale" and the other vector (***R***) represents "incremental distance". Length scale is associated with some property of a "material object". The vectors are defined as;

$$\boldsymbol{r} = r_1\boldsymbol{e_{11}} + r_2\boldsymbol{e_{12}} + r_3\boldsymbol{e_{13}} + r_4\boldsymbol{e_{14}}$$

$$\boldsymbol{R} = \partial R_1\boldsymbol{e_{21}} + \partial R_2\boldsymbol{e_{22}} + \partial R_3\boldsymbol{e_{23}} + \partial R_4\boldsymbol{e_{24}}$$

Where; $\boldsymbol{e_{n1}}$, $\boldsymbol{e_{n2}}$, $\boldsymbol{e_{n3}}$, $\boldsymbol{e_{n4}}$ are directional vectors (unit vectors in 4D)

Each set (n) of unit vectors is a unique frame of reference (n = 1,2)

r_1 , r_2 , r_3 , r_4 are components of length scale

∂R_1 , ∂R_2 , ∂R_3 , ∂R_4 are components of incremental displacement

The vectors have magnitude; $|\boldsymbol{r}| = r_5$

$|\boldsymbol{R}| = \partial R_5$

The components are related to magnitudes; $r_1^2 + r_2^2 + r_3^2 + r_4^2 = r_5^2$

$\partial R_1^2 + \partial R_2^2 + \partial R_3^2 + \partial R_4^2 = \partial R_5^2$

Sub-components (r_6 , r_7 , r_8) and (∂R_6 , ∂R_7 , ∂R_8) are related as;

$r_6^2 = r_5^2 - r_4^2 = r_7^2 + r_3^2$ $\quad$ $\partial R_6^2 = \partial R_5^2 - \partial R_4^2 = \partial R_7^2 + \partial R_3^2$

$r_7^2 = r_6^2 - r_3^2 = r_1^2 + r_2^2$ $\quad$ $\partial R_7^2 = \partial R_6^2 - \partial R_3^2 = \partial R_1^2 + \partial R_2^2$

$r_8^2 = r_5^2 - r_7^2 = r_3^2 + r_4^2$ $\quad$ $\partial R_8^2 = \partial R_5^2 - \partial R_7^2 = \partial R_3^2 + \partial R_4^2$

Note; $r_5^2 = r_6^2 + r_4^2 = r_7^2 + r_8^2$

Angular geometry;

Each vector contains four "component angles" (A_{n1}, A_{n2}, A_{n3}, A_{n4}).

The component angles of length scale (n=1) have angular geometry;

$r_1 = r_7\cos(A_{11})$ and; $r_2 = r_7\sin(A_{11})$

$r_7 = r_6\cos(A_{12})$ and; $r_3 = r_6\sin(A_{12})$

$r_6 = r_5\cos(A_{13})$ and; $r_4 = r_5\sin(A_{13})$

$r_3 = r_8\cos(A_{14})$ and; $r_4 = r_8\sin(A_{14})$

The component angles of incremental displacement (n=2) have angular geometry;

$\partial R_1 = \partial R_7\cos(A_{21})$ and; $\partial R_2 = \partial R_7\sin(A_{21})$

$\partial R_7 = \partial R_6\cos(A_{22})$ and; $\partial R_3 = \partial R_6\sin(A_{22})$

$\partial R_6 = \partial R_5\cos(A_{23})$ and; $\partial R_4 = \partial R_5\sin(A_{23})$

$\partial R_3 = \partial R_8\cos(A_{24})$ and; $\partial R_4 = \partial R_8\sin(A_{24})$

Interaction;

Components of the spatial regions and the displacement vectors may interact using angular geometry. The angular interaction rules are;

Rule 1; $\text{Cos}^2(A_{12}) = \text{Tan}(\beta)$

Rule 2; $2\text{Sin}^2(A_{21}) = \text{Sin}(2\beta)$

Rule 3; $2\text{Sin}^2(A_{23}) = \text{Sin}(2\beta)$

The rules may be re-written as follows;

Rule 2; $2(\partial R_2/\partial R_7)^2 = 2r_6^2r_7^2/(r_6^4 + r_7^4)$

Giving; $\partial R_2^2(r_6^4 + r_7^4)/(r_6^2r_7^2) = \partial R_7^2$

$\partial R_2^2(r_6^2/r_7^2) + \partial R_2^2(r_7^2/r_6^2) = \partial R_7^2$

Rule 3; $2(\partial R_4/\partial R_5)^2 = 2r_6^2r_7^2/(r_6^4 + r_7^4)$

Giving; $\partial R_4^2(r_6^4 + r_7^4)/(r_6^2r_7^2) = \partial R_5^2$

$\partial R_4^2(r_6^2/r_7^2) + \partial R_4^2(r_7^2/r_6^2) = \partial R_5^2$

The General Metric;

The initial metric is; $\partial R_8^2 = \partial R_5^2 - \partial R_7^2$

The general metric combines information from spherical regions and displacement vectors;

$$\partial R_8^2 = \partial R_4^2(r_6^2/r_7^2) + \partial R_4^2(r_7^2/r_6^2) - \partial R_2^2(r_6^2/r_7^2) - \partial R_2^2(r_7^2/r_6^2)$$

The general metric is; $\partial R_8^2 - \partial R_4^2(r_6^2/r_7^2) + \partial R_2^2(r_6^2/r_7^2) + \partial R_2^2(r_7^2/r_6^2) = \partial R_4^2(r_7^2/r_6^2)$

Where; $r_5^2 = r_6^2 + r_4^2 = r_7^2 + r_8^2$

$r_7^2 = r_6^2 + r_4^2 - r_8^2$

Standard Nomenclature;

The components of the general metric may be expressed as dimensions of space-time, or length scales of matter;

$r_6 = r$ and; $r_4 = r_Q$ and; $r_8 = (rr_S)^{½}$

$r_7^2 = r_6^2 + r_4^2 - r_8^2 = r^2 + r_Q^2 - rr_S$

$\partial R_8 = c\partial T$ and; $\partial R_4 = c\partial t$ and; $\partial R_2 = \partial r$

The general metric may be written as;

$$c^2\partial T^2 - c^2\partial t^2(r^2/r_7^2) + \partial r^2(r^2/r_7^2) + \partial r^2(r_7^2/r^2) = c^2\partial t^2(r_7^2/r^2)$$

Components may be related to incremental arc length (∂U_θ);

$c = i(r_7/r)\partial U_\theta/\partial t$ giving; $c^2 + (r_7^2/r^2)\partial U_\theta^2/\partial t^2 = 0$

Where; i is the complex constant

This may be represented as reciprocal forces; $mc^2/r + m(r_7^2/r^3)\partial U_\theta^2/\partial t^2 = 0$

$$mc^2/r + mv_\theta^2/r = 0$$

Components may also be related to incremental arc length (∂U_ϕ); $r_7/r = \partial U_\phi/\partial r$

If incremental arc lengths are included, the general metric may be written as;

$$c^2\partial T^2 + \partial U_\theta^2 + \partial r^2(r^2/r_7^2) + \partial U_\phi^2 = c^2\partial t^2(r_7^2/r^2)$$

$$c^2\partial T^2 + r^2\partial\theta^2 + \partial r^2(r^2/r_7^2) + w^2\partial\phi^2 = c^2\partial t^2(r_7^2/r^2)$$

The Reissner-Nordström Metric:

The general metric may be re-written as the Reissner-Nordström metric;

$$c^2\partial T^2 + r^2\partial\theta^2 + \partial r^2(r^2/r_7^2) + r^2\mathrm{Sin}^2(\theta)\partial\phi^2 = c^2\partial t^2(r_7^2/r^2)$$

Where; $r_7^2/r^2 = (r^2 + r_Q^2 - rr_S)/r^2 = 1 + r_Q^2/r^2 - r_S/r$

Conclusion;

Two types of displacement are “length scale” and “incremental distance”. Vectors representing each type act upon a common point which is also the center of three spherical regions. Component relationships will give the Reissner-Nordström metric.