10 Acceleration

The Reissner-Nordström metric may be simply obtained from ratios of “acceleration forms”.

Forms of Representation;

Acceleration may be represented in different forms;

as a scalar; g

as vectors; $\boldsymbol{u}$, $\boldsymbol{v}$

as a tensor; $\boldsymbol{G}$

The vectors are four dimensional. The tensor is rank two.

Vectors;

The 4D vectors of acceleration are;

$\boldsymbol{u} = \Sigma_j(u_j \boldsymbol{e}_{uj})$ (j = 1,2,3,4)

$\boldsymbol{v} = \Sigma_n(v_n \boldsymbol{e}_{vn})$ (n = 1,2,3,4)

Where; u_j , v_n are vector components

$\boldsymbol{e}_{uj}$, $\boldsymbol{e}_{vn}$ are unit vectors (direction vectors)

Each vector has a unique basis ($\boldsymbol{e}_u$, $\boldsymbol{e}_v$)

Acceleration Ratios;

The different forms of acceleration may be related ratios;

The “Primary Ratio Equation” is;	$\boldsymbol{u}/\boldsymbol{v} = \boldsymbol{G}/g$	Giving products;	$g\boldsymbol{u} = \boldsymbol{G}\boldsymbol{v}$
The “Secondary Ratio Equation” is;	$\boldsymbol{w}_n/\boldsymbol{e}_{vn} = \boldsymbol{G}/e_0$	Giving products;	$e_0\boldsymbol{w}_n = \boldsymbol{G}\boldsymbol{e}_{vn}$

Basis Ratios;

The “Basis Ratio Equation” is; $e_0/e_{0nj} = \boldsymbol{e}_{uj}/\boldsymbol{e}_{wnj}$ Giving; $e_0\boldsymbol{e}_{wnj} = e_{0nj}\boldsymbol{e}_{uj}$

Where; e_{0nj} is a “Kronecker scalar”;

If; n = J Then; $e_{0nj} = 1$

If; n ≠ J Then; $e_{0nj} = 0$

The Acceleration Equation;

The "Primary Ratio Equation" is; $\boldsymbol{u}/\boldsymbol{v} = \boldsymbol{G}/g$

Rearranging gives products; $g\boldsymbol{u} = \boldsymbol{G}\boldsymbol{v}$

Expanding $\boldsymbol{v}$; $g\boldsymbol{u} = \boldsymbol{G}(v_1\boldsymbol{e}_{v1} + v_2\boldsymbol{e}_{v2} + v_3\boldsymbol{e}_{v3} + v_4\boldsymbol{e}_{v4})$

Primary acceleration equation; $g\boldsymbol{u} = v_1\boldsymbol{G}\boldsymbol{e}_{v1} + v_2\boldsymbol{G}\boldsymbol{e}_{v2} + v_3\boldsymbol{G}\boldsymbol{e}_{v3} + v_4\boldsymbol{G}\boldsymbol{e}_{v4}$

The "Secondary Ratio Equation" is; $\boldsymbol{w}_n/\boldsymbol{e}_{vn} = \boldsymbol{G}/e_0$

Rearranging gives; $e_0\boldsymbol{w}_n = \boldsymbol{G}\boldsymbol{e}_{vn}$

Acceleration products become; $g\boldsymbol{u} = v_1e_0\boldsymbol{w}_1 + v_2e_0\boldsymbol{w}_2 + v_3e_0\boldsymbol{w}_3 + v_4e_0\boldsymbol{w}_4$

Expanding $\boldsymbol{w}_n$;

$$\begin{aligned} g\boldsymbol{u} = \; & v_1e_0(w_{11}\boldsymbol{e}_{w11} + w_{12}\boldsymbol{e}_{w12} + w_{13}\boldsymbol{e}_{w13} + w_{14}\boldsymbol{e}_{w14}) + \\ & v_2e_0(w_{21}\boldsymbol{e}_{w21} + w_{22}\boldsymbol{e}_{w22} + w_{23}\boldsymbol{e}_{w23} + w_{24}\boldsymbol{e}_{w24}) + \\ & v_3e_0(w_{31}\boldsymbol{e}_{w31} + w_{32}\boldsymbol{e}_{w32} + w_{33}\boldsymbol{e}_{w33} + w_{34}\boldsymbol{e}_{w34}) + \\ & v_4e_0(w_{41}\boldsymbol{e}_{w41} + w_{42}\boldsymbol{e}_{w42} + w_{43}\boldsymbol{e}_{w43} + w_{44}\boldsymbol{e}_{w44}) \end{aligned}$$

Change of Basis;

A change of basis is required.

The "Basis Ratio Equation" is; $e_0/e_{0nj} = \boldsymbol{e}_{uj}/\boldsymbol{e}_{wnj}$

Rearranging gives; $e_0\boldsymbol{e}_{wnj} = e_{0nj}\boldsymbol{e}_{uj}$

Acceleration products may be written as;

$$\begin{aligned} g\boldsymbol{u} = \; & v_1(w_{11}e_{011}\boldsymbol{e}_{u1} + w_{12}e_{012}\boldsymbol{e}_{u2} + w_{13}e_{013}\boldsymbol{e}_{u3} + w_{14}e_{014}\boldsymbol{e}_{u4}) + \\ & v_2(w_{21}e_{021}\boldsymbol{e}_{u1} + w_{22}e_{022}\boldsymbol{e}_{u2} + w_{23}e_{023}\boldsymbol{e}_{u3} + w_{24}e_{024}\boldsymbol{e}_{u4}) + \\ & v_3(w_{31}e_{031}\boldsymbol{e}_{u1} + w_{32}e_{032}\boldsymbol{e}_{u2} + w_{33}e_{033}\boldsymbol{e}_{u3} + w_{34}e_{034}\boldsymbol{e}_{u4}) + \\ & v_4(w_{41}e_{041}\boldsymbol{e}_{u1} + w_{42}e_{042}\boldsymbol{e}_{u2} + w_{43}e_{043}\boldsymbol{e}_{u3} + w_{44}e_{044}\boldsymbol{e}_{u4}) \end{aligned}$$

Applying Kronecker values;

$$\begin{aligned} g\boldsymbol{u} = \; & v_1(w_{11}\boldsymbol{e}_{u1} + 0 + 0 + 0) + \\ & v_2(0 + w_{22}\boldsymbol{e}_{u2} + 0 + 0) + \\ & v_3(0 + 0 + w_{33}\boldsymbol{e}_{u3} + 0) + \\ & v_4(0 + 0 + 0 + w_{44}\boldsymbol{e}_{u4}) \end{aligned}$$

giving acceleration products in $\boldsymbol{e}_u$; $g\boldsymbol{u} = v_1w_{11}\boldsymbol{e}_{u1} + v_2w_{22}\boldsymbol{e}_{u2} + v_3w_{33}\boldsymbol{e}_{u3} + v_4w_{44}\boldsymbol{e}_{u4}$

Scalar Acceleration;

Expanding $\boldsymbol{u}$; $g(u_1\boldsymbol{e}_{u1} + u_2\boldsymbol{e}_{u2} + u_3\boldsymbol{e}_{u3} + u_4\boldsymbol{e}_{u4}) = v_1w_{11}\boldsymbol{e}_{u1} + v_2w_{22}\boldsymbol{e}_{u2} + v_3w_{33}\boldsymbol{e}_{u3} + v_4w_{44}\boldsymbol{e}_{u4}$

Relating components; $gu_j = v_jw_{jj}$

The combined scalar product is the sum of all acceleration product terms;

$$g(u_1 + u_2 + u_3 + u_4) = v_1w_{11} + v_2w_{22} + v_3w_{33} + v_4w_{44}$$

Assume; $u_1 + u_2 + u_3 + u_4 = u_5$

The scalar acceleration product is; $gu_5 = v_1w_{11} + v_2w_{22} + v_3w_{33} + v_4w_{44}$

Definition of Scalar Acceleration;

Each scalar of acceleration (g_N) is; $g_N = v_N^2/r_N$

Where; v_N is velocity

r_N is displacement

The scalar acceleration product is; $gu_5 = v_1w_{11} + v_2w_{22} + v_3w_{33} + v_4w_{44}$

Substituting definitions of acceleration;

$$(v_0^2/r_0)(v_{u5}^2/r_{u5}) = (v_{v1}^2/r_{v1})(v_{w11}^2/r_{w11}) + (v_{v2}^2/r_{v2})(v_{w22}^2/r_{w22}) + (v_{v3}^2/r_{v3})(v_{w33}^2/r_{w33}) + (v_{v4}^2/r_{v4})(v_{w44}^2/r_{w44})$$

Special Conditions;

Special conditions may apply to displacement; $r_0 = r_{u5} = r_{w11} = r_{v3} = r_{v4} = r$

$r_{v1} = r_{v2} = r_{w22} = r_{w33} = r_{w44} = R$

Where; $r^2 = x^2 + y^2 + z^2$

$R = r - r_S - r_Q^2/r$

r_S is the Schwarzschild radius

r_Q is the electric length-scale

Special conditions may also apply to velocity; $v_{u5} = v_{w22} = v_{w33} = v_{w44} = v_\lambda$

Giving the special acceleration product;

$$(v_0^2/r)(v_\lambda^2/r) = (v_{v1}^2/R)(v_{w11}^2/r) + (v_{v2}^2/R)(v_\lambda^2/R) + (v_{v3}^2/r)(v_\lambda^2/R) + (v_{v4}^2/r)(v_\lambda^2/R)$$

Re-arranging; $(R/r)v_0^2v_\lambda^2 = v_{v1}^2v_{w11}^2 + (r/R)v_{v2}^2v_\lambda^2 + v_{v3}^2v_\lambda^2 + v_{v4}^2v_\lambda^2$

Velocity Definitions;

Velocities may be defined as;

$v_0 = v_{v1} = c$ $\quad v_\lambda = \partial\lambda\partial f = \partial\lambda/\partial T$ $\quad v_{w11} = \partial\lambda/\partial t$

$v_{v2} = \partial r/\partial t$ $\quad v_{v3} = \partial U_\theta/\partial t$ $\quad v_{v4} = \partial U_\phi/\partial t$

Substituting for velocities gives;

$$(R/r)c^2(\partial\lambda^2/\partial T^2) = c^2(\partial\lambda^2/\partial t^2) + (r/R)(\partial r^2/\partial t^2)(\partial\lambda^2/\partial T^2) + (\partial U_\theta^2/\partial t^2)(\partial\lambda^2/\partial T^2) + (\partial U_\phi^2/\partial t^2)(\partial\lambda^2/\partial T^2)$$

$$(R/r)c^2(1/\partial T^2) = c^2(1/\partial t^2) + (r/R)(\partial r^2/\partial t^2)(1/\partial T^2) + (\partial U_\theta^2/\partial t^2)(1/\partial T^2) + (\partial U_\phi^2/\partial t^2)(1/\partial T^2)$$

Giving the special metric;

$$(R/r)c^2\partial t^2 = c^2\partial T^2 + (r/R)\partial r^2 + \partial U_\theta^2 + \partial U_\phi^2$$

Angular Geometry;

Dimensional components are; $w^2 = r^2 - z^2 = x^2 + y^2$

$x = w\mathrm{Cos}(\phi)$ and; $y = w\mathrm{Sin}(\phi)$

$z = r\mathrm{Cos}(\theta)$ and; $w = r\mathrm{Sin}(\theta)$

Angles (θ , ϕ) are defined as; $\theta = U_\theta/r$ and; $\phi = U_\phi/w$

$r\partial\theta = \partial U_\theta$ and; $w\partial\phi = \partial U_\phi$

Substituting for angular geometry gives the angular metric;

$$(R/r)c^2\partial t^2 = c^2\partial T^2 + (r/R)\partial r^2 + r^2\partial\theta^2 + w^2\partial\phi^2$$

The Reissner-Nordström Metric;

Substituting for 'w' gives the Reissner-Nordström metric;

$$(R/r)c^2\partial t^2 = c^2\partial T^2 + (r/R)\partial r^2 + r^2\partial\theta^2 + r^2\mathrm{Sin}^2(\theta)\partial\phi^2$$

Where; $R = r - r_S - r_Q^2/r$

Conclusion;

The Reissner-Nordström metric may be simply obtained from ratios of forms of acceleration.