08 Particle Interaction

Two particles interact. Assume that;

- each particle has momentum
- the momentum of each particle changes with time
- momentum is exchanged during the interaction
- the interaction is elastic (each particle recoils at some angle)
- total momentum is conserved.

Momentum may be represented as a vector. The interaction may be represented as a set of four momentum vectors. The vectors may be grouped into time phases; an initial phase and a final phase, representing the beginning and end of the interaction.

Each phase has two vectors, a vector represents the "phased momentum" of each particle. Vector magnitudes will give the conservation rule. Three rules govern the interaction.

Scalars of phased momentum may correspond to a Feynman diagram.

An interaction of special interest is between an electron and a photon as represented by the Compton equation, published in 1923. Energy is transferred from the photon to the electron. The collision is elastic and each particle recoils in a different direction. Vector magnitudes will give the Compton equation. Mass dilation and De Broglie momentum may also be included in the Compton equation.

An interaction may also be represented as an "interaction vector" of momentum.

Vector Phases;

Two particles (Q_A , Q_B) interact. The interaction is represented by four vectors of "phased momentum" ($\boldsymbol{p}_{a1}$, $\boldsymbol{p}_{b1}$, $\boldsymbol{p}_{a2}$, $\boldsymbol{p}_{b2}$). The four vectors of momentum are grouped into two time phases;

Phase 1; ($\boldsymbol{p}_{a1}$, $\boldsymbol{p}_{b1}$)

Phase 2; ($\boldsymbol{p}_{a2}$, $\boldsymbol{p}_{b2}$)

Where; $\boldsymbol{p}_{a1}$ is the initial momentum of one particle (Q_A)

$\boldsymbol{p}_{b1}$ is the initial momentum of the other particle (Q_B)

$\boldsymbol{p}_{a2}$ is the final momentum of one particle (Q_A)

$\boldsymbol{p}_{b2}$ is the final momentum of the other particle (Q_B)

Vector Magnitudes;

Each vector has a magnitude; $|\boldsymbol{p_{an}}| = p_{an}$

$|\boldsymbol{p_{bn}}| = p_{bn}$

Where; n is the phase number (n = 1,2)

Interaction Rules;

Three rules govern the interaction;

Rule 1; (The "Conservation Rule") is; $p_{a1} + p_{b1} = p_{a2} + p_{b2}$

Rule 2; (The "Product Rule")is; $p_{a1}p_{b1} = p_{a2}p_{b2}$

Rule 3; (The "Recoil Rule") is; $p_{a1} = p_{a2}Cos(\theta_a)$ and; $p_{b2} = p_{b1}Cos(\theta_b)$

An "elastic collision" allows a particle to recoil. The angle of recoil (scattering angle) is; θ_b

Momentum is conserved over an interaction giving the conservation rule (summation rule).

The Interaction Equation;

The Conservation Rule is; $p_{a1} + p_{b1} = p_{a2} + p_{b2}$

Re-arranging; $p_{b1} - p_{b2} = p_{a2} - p_{a1}$

$1 - p_{b2}/p_{b1} = p_{a1}[p_{a2}/(p_{a1}p_{b1}) - 1/p_{b1}]$

The product rule gives; $1 - p_{b2}/p_{b1} = p_{a1}[1/p_{b2} - 1/p_{b1}]$

The interaction equation is; $1 - Cos(\theta_b) = p_{a1}[1/p_{b2} - 1/p_{b1}]$

The Compton Equation;

An interaction of special interest is between an electron and a high energy photon as represented by the Compton equation, published in 1923. Energy is transferred from the photon to the electron. The collision is elastic and each particle recoils in a different direction. Definitions of momentum will relate the interaction equation to the Compton equation.

Where; Q_A is the electron

Q_B is the photon

Magnitudes of momentum are defined as;

$p_{b1} = h/\lambda_{b1}$ (initial momentum of photon)

$p_{b2} = h/\lambda_{b2}$ (final momentum of photon)

$p_{a1} = m_1c$ (initial momentum of electron)

p_{a2} (final momentum of electron)

The interaction equation is; $1 - Cos(\theta_b) = p_{a1}[1/p_{b2} - 1/p_{b1}]$

Substituting for momentum gives; $1 - Cos(\theta_b) = m_1c[\lambda_{b2}/h - \lambda_{b1}/h]$

The Compton equation may be written as; $(1 - Cos\theta_b)(h/m_1c) = \lambda_{b2} - \lambda_{b1}$

Where; θ_b is the scattering angle

λ_{b1} is the initial wavelength of the photon

λ_{b2} is the wavelength of photon after scattering ($\lambda_{b2} > \lambda_{b1}$)

m_1 is electron rest mass

h is the Plank constant

c is the light constant

Mass Dilation;

A "phase span" (p_{a3}) links both phases of momentum (p_{a1} , p_{a2}) for Q_A; $p_{a3} = (p_{a2}^2 - p_{a1}^2)^{½}$

Rearranging; $p_{a1}^2/p_{a2}^2 = 1 - p_{a3}^2/p_{a2}^2$

Definitions of phased momentum are;

$p_{a1} = m_1c$ (initial momentum of electron)

$p_{a2} = m_2c$ (final momentum of electron)

$p_{a3} = m_2v_3$ (phase span of electron)

Substitution for momentum resembles mass dilation; $m_1^2/m_2^2 = 1 - v_3^2/c^2$

De Broglie Momentum;

A "phase span" (p_{b3}) links both phases of momentum (p_{b1} , p_{b2}) for Q_B; $p_{b3} = (p_{b1}^2 - p_{b2}^2)^{½}$

Definitions of phased momentum are;

$p_{b1} = h/\lambda_1$ (initial momentum of photon)

$p_{b2} = h/\lambda_2$ (final momentum of photon)

$p_{b3} = h/\lambda_3$ (phase span of photon)

The phase spans are equal; $p_{b3} = p_{a3}$

Substitution for momentum resembles De Broglie momentum; $h/\lambda_3 = m_2v_3$

Total Phase Span;

The total span (P_T) of an interaction connects both phases of both particles (p_{a1}, p_{a2}, p_{b1}, p_{b2}). The total span of an interaction also connects the "phase points" of each particle.

The phase point for Q_A is the intersection (X); $p_{a1}Xp_{a2}$

The phase point for Q_B is the intersection (X); $p_{b1}Xp_{b2}$

The interaction resembles a Feynman diagram; $(p_{a1}Xp_{a2})$–"p_T"–$(p_{b1}Xp_{b2})$

Where; -"P_T"- is read; "is connected by P_T"

Total span is; $p_T^2 = p_{a1}^2 + p_{b1}^2 = p_{a2}^2 + p_{b2}^2$

Total span relates interaction rule 1 and rule 2;

Rule 1 gives; $p_{a1} + p_{b1} = p_{a2} + p_{b2}$

$(p_{a1} + p_{b1})^2 = (p_{a2} + p_{b2})^2$

$p_{a1}^2 + p_{b1}^2 + 2p_{a1}p_{b1} = p_{a2}^2 + p_{b2}^2 + 2p_{a2}p_{b2}$

$p_T^2 + 2p_{a1}p_{b1} = p_T^2 + 2p_{a2}p_{b2}$

giving Rule 2; $p_{a1}p_{b1} = p_{a2}p_{b2}$

The total span of an interaction is also the magnitude of an "interaction vector".

The Interaction Vector;

The interaction may be represented as a vector of momentum ($\boldsymbol{p}$) having a magnitude which represents the "connection" between both phases of both particles. The vector magnitude is "total phase span".

The interaction vector is; $\boldsymbol{p} = p_1\boldsymbol{e_1} + p_2\boldsymbol{e_2} + p_3\boldsymbol{e_3}$

Where; $\boldsymbol{e_1}$, $\boldsymbol{e_2}$, $\boldsymbol{e_3}$ are direction vectors (unit vectors)

p_1, p_2, p_3 are components of momentum

The vector has magnitude; $|\boldsymbol{p}| = p_4$

Components are related to magnitude; $p_1^2 + p_2^2 + p_3^2 = p_4^2$

Sub-components (p_5 , p_6) are; $p_5^2 = p_1^2 + p_2^2 = p_4^2 - p_3^2$

$p_6^2 = p_2^2 + p_3^2 = p_4^2 - p_1^2$

Angular sub-components (θ_a, θ_b) have trig ratios;

$p_3 = p_6 Cos(\theta_a)$ and; $p_2 = p_6 Sin(\theta_a)$

$p_1 = p_5 Cos(\theta_b)$ and; $p_2 = p_5 Sin(\theta_b)$

Component Definitions;

The components of the interaction vector may be related to the scalars of phased momentum;

$p_1 = p_{b2}$

$p_2 = p_{a3} = p_{b3}$

$p_3 = p_{a1}$

$p_4 = p_T$

$p_5 = p_{b1}$

$p_6 = p_{a2}$

Conclusion;

Particle interaction may be represented as an "interaction vector" or as phased scalars of momentum. Three rules govern the interaction.

The Compton equation is a unique interaction which may be represented by the interaction vector and by phased momentum.

A Feynman diagram may be constructed from phased momentum.