36 Gravitational Change Factors

The Lorentz factor defines a change to the properties of an object in motion. It may be called a “change factor”.

It is also useful to define an “incremental change factor” (a first differential).

A gravitational field may be represented as a field vector. The field may change, and the vector magnitude changes also. The change in magnitude may include a “change factor” such as the Lorentz factor.

Assume a field changes twice, and two change factors are required (one for each change). The change factors may be;

- the Lorentz factor

- the incremental factor

Combination of the change factors will give a complete transform and lead to the Schwarzschild metric.

Change Factors;

Two factors of change are;

Lorentz factor (γ); $\gamma = 1/(1 - v^2/c^2)^{½}$

incremental factor (δ); $\delta = \partial n/\partial R$

Field Changes;

A gravitational field vector (***g***) changes twice.

The first change has an initial magnitude (g_0) and a final magnitude (g_1).

The second change continues from the first, having has an initial magnitude (g_1) and a final magnitude (g_2).

The first change applies the Lorentz factor; $g_1 = \gamma g_0$

The second change applies the incremental factor; $g_2 = \delta g_1$

Sequential Field Definitions;

The field magnitudes (in sequence) are; $g_0 = Gm/R_0^2$

$g_1 = Gm/R_1^2$

$g_2 = Gm/R_2^2$

The Equation of Change;

The equation of change includes both change factors. The first change may be represented as; $g_1 = \gamma g_0$

Giving; $g_0^2/g_1^2 + v^2/c^2 = 1$

The second change gives; $\delta^2 g_0^2/g_2^2 + v^2/c^2 = 1$

Substituting for magnitudes; $\delta^2(G^2m^2/R_0^4)/(G^2m^2/R_2^4) + v^2/c^2 = 1$

Giving the equation of change; $(\partial n^2/\partial R^2)(R_2^4/R_0^4) + v^2/c^2 = 1$

Vibration;

Assume the field has vibration represented as frequency (f) and incremental frequency (∂f). It is convenient to use inverse frequency (t) and incremental time (∂t).

Displacement;

Assume four instances of displacement (r, R, s, u). Each displacement is the length of a line segment. They may be arranged as two groups; straight line segments (r, R) and curved line segments (s, u).

Where; r is a radial displacement

$R = r - r_S$

r_S is the Schwarzschild radius

s is the segment length of an undefined curve

u is the length of a circular segment, the arc length of an angle (Ω); $u = r\Omega$

Incremental Displacement;

Assume four instances of incremental displacement (∂r, ∂R, ∂s, ∂u). They may be arranged as two groups; straight increments (∂r, ∂R) and curved increments (∂s, ∂u).

Where; $\partial R = c\partial t$

$\partial u = r\partial\Omega$

The angular incremental (∂u) has two components (∂u_θ, ∂u_ϕ); $\partial u^2 = \partial u_\theta^2 + \partial u_\phi^2$

Where; $\partial u_\theta = r\partial\theta$

$\partial u_\phi = w\partial\phi$ and; $w = r\mathrm{Sin}(\theta)$

The curved increments may be combined, forming the "total curved increment" (∂n); $\partial n^2 = \partial s^2 + \partial u^2$

The change from total curved increment (∂n) to straight increment (∂R) is given by the incremental change factor (δ);

$$\partial n = \delta\partial R$$

Velocity;

Velocity may be represented as a vector. Two vectors of velocity ($\boldsymbol{v_1}$, $\boldsymbol{v_2}$) have parts (v_{1N}, v_{2N}) defined as;

$v_{14} = v$	$v_{15} = r/t$	$v_{16} = R_2/t$	$v_{17} = R_0/t$
$v_{22} = \partial r/\partial t$	$v_{25} = r/t$	$v_{26} = R_0/t$	$v_{27} = R/t$

Where; v is Lorentz velocity

R_0, R_2 are sequential field distances

r, R are displacements

∂r is incremental displacement

∂t is incremental time

The vectors are "in agreement"; $|\boldsymbol{v_1}| = |\boldsymbol{v_2}|$

Giving; $v_{15} = v_{25}$

Velocity ratios give three rules;

Rule 1; $v_{16}/v_{15} = v_{17}/v_{16}$ giving; $v_{16}^2 = v_{15}v_{17}$ giving; $R_2^2 = rR_0$

Rule 2; $v_{14}/v_{15} = v_{22}/v_{27}$

Rule 3; $v_{26}/v_{25} = v_{27}/v_{26}$ giving; $v_{26}^2 = v_{25}v_{27}$ giving; $R_0^2 = rR$

The Velocity Equation;

The equation of change is; $(\partial n^2/\partial R^2)(R_2^4/R_0^4) + v^2/c^2 = 1$

Velocity substitution gives; $(\partial n^2/\partial R^2)(t^4 v_{16}^4/t^4 v_{17}^4) + v_{14}^2/c^2 = 1$

$$(\partial n^2/\partial R^2)(v_{16}^4/v_{17}^4) + v_{14}^2/c^2 = 1$$

Rule 1 gives; $(\partial n^2/\partial R^2)(v_{15}{}^2/v_{17}{}^2) + v_{14}{}^2/c^2 = 1$

$v_{17} = v_{26}$ giving; $(\partial n^2/\partial R^2)(v_{15}{}^2/v_{26}{}^2) + v_{14}{}^2/c^2 = 1$

Rule 2 gives; $(\partial n^2/\partial R^2)(v_{15}{}^2/v_{26}{}^2) + (v_{15}{}^2/v_{27}{}^2)(v_{22}{}^2/c^2) = 1$

Rule 3 gives; $(\partial n^2/\partial R^2)(v_{15}{}^2/v_{25}v_{27}) + (v_{15}{}^2/v_{27}{}^2)(v_{22}{}^2/c^2) = 1$

$v_{15} = v_{25}$ giving; $(\partial n^2/\partial R^2)(v_{15}/v_{27}) + (v_{15}{}^2/v_{27}{}^2)(v_{22}{}^2/c^2) = 1$

The velocity equation is; $(\partial n^2/\partial R^2) + (v_{15}/v_{27})(v_{22}{}^2/c^2) = v_{27}/v_{15}$

The Metric of Change;

The velocity equation is; $(\partial n^2/\partial R^2) + (v_{15}/v_{27})(v_{22}{}^2/c^2) = v_{27}/v_{15}$

Velocity definitions give; $(\partial n^2/\partial R^2) + (r/R)(v_{22}{}^2/c^2) = R/r$

$(\partial n^2/\partial R^2) + (r/R)(\partial r^2/c^2\partial t^2) = R/r$

$(\partial n^2/\partial R^2)(c^2\partial t^2) + (r/R)\partial r^2 = (R/r)c^2\partial t^2$

Giving the "metric of change"; $\partial n^2 + (r/R)\partial r^2 = (R/r)c^2\partial t^2$

The Schwarzschild Metric;

The "metric of change" is; $\partial n^2 + (r/R)\partial r^2 = (R/r)c^2\partial t^2$

Curved increments give; $\partial s^2 + \partial u^2 + (r/R)\partial r^2 = (R/r)c^2\partial t^2$

Giving the Schwarzschild metric; $\partial s^2 + r^2\partial\Omega^2 + (r/R)\partial r^2 = (R/r)c^2\partial t^2$

Where; $R = r - r_S$

r_S is the Schwarzschild radius

$\partial\Omega^2 = \partial\theta^2 + \mathrm{Sin}^2(\theta)\partial\phi^2$

The Velocity Vectors;

Two 4D vectors of velocity ($\boldsymbol{v_n}$) are; $\boldsymbol{v_n} = v_{n1}\boldsymbol{e_{n1}} + v_{n2}\boldsymbol{e_{n2}} + v_{n3}\boldsymbol{e_{n3}} + v_{n4}\boldsymbol{e_{n4}}$

Where; n is the vector identifier (n = 1,2)

$\boldsymbol{e_{n1}}$, $\boldsymbol{e_{n2}}$, $\boldsymbol{e_{n3}}$, $\boldsymbol{e_{n4}}$ are directional vectors (unit vectors)

the set ($\boldsymbol{e_{n1}}$, $\boldsymbol{e_{n2}}$, $\boldsymbol{e_{n3}}$, $\boldsymbol{e_{n4}}$) is a frame of reference (each vector has a unique frame of reference)

v_{n1} , v_{n2} , v_{n3} , v_{n4} are components of velocity

Each vector has a magnitude; $|\boldsymbol{v_n}| = v_{n5}$

Magnitude is related to components; $v_{n1}^2 + v_{n2}^2 + v_{n3}^2 + v_{n4}^2 = v_{n5}^2$

Sub-components (v_{n6}, v_{n7}) are; $v_{n6}^2 = v_{n5}^2 - v_{n4}^2 = v_{n7}^2 + v_{n3}^2$

$v_{n7}^2 = v_{n1}^2 + v_{n2}^2 = v_{n6}^2 - v_{n3}^2$

Angular Components;

Each vector has angular components (A_{n1}, A_{n2}, A_{n3},) with corresponding geometry;

$v_{n1} = v_{n7}Cos(A_{n1})$ $v_{n2} = v_{n7}Sin(A_{n1})$

$v_{n71} = v_{n6}Cos(A_{n2})$ $v_{n3} = v_{n6}Sin(A_{n2})$

$v_{n6} = v_{n5}Cos(A_{n3})$ $v_{n4} = v_{n5}Sin(A_{n3})$

Three rules apply to the velocity vectors;

Rule 1; $A_{12} = A_{13}$ giving; $Cos(A_{12}) = Cos(A_{13})$ and; $v_{17}/v_{16} = v_{16}/v_{15}$

Rule 2; $A_{13} = A_{21}$ giving; $Sin(A_{13}) = Sin(A_{21})$ and; $v_{14}/v_{15} = v_{22}/v_{27}$

Rule 3; $A_{22} = A_{23}$ giving; $Cos(A_{22}) = Cos(A_{23})$ and; $v_{27}/v_{26} = v_{26}/v_{25}$

Conclusion;

Two types of change factors are the Lorentz factor and an incremental change ratio. Combination of the change factors will lead to the Schwarzschild metric.