37 Radiant Forces

An active emitter shines light and heat into local space. Radiant forces combine to produce radiant brightness. A force may be associated with the emitter, and another force may be associated with the radiation.

Each force may be represented as a vector. Each vector is four dimensional. If three "radiant conditions" apply, then the components of force will give the Stephan-Boltzmann equation of brightness.

The Radiant Vectors;

One vector of force ($\boldsymbol{F_1}$) may be associated with an emitter, and another vector of force ($\boldsymbol{F_2}$) may be associated with radiation. The force vectors are;

$$\boldsymbol{F_1} = F_{11}\boldsymbol{e_{11}} + F_{12}\boldsymbol{e_{12}} + F_{13}\boldsymbol{e_{13}} + F_{14}\boldsymbol{e_{14}}$$

$$\boldsymbol{F_2} = F_{21}\boldsymbol{e_{21}} + F_{22}\boldsymbol{e_{22}} + F_{23}\boldsymbol{e_{23}} + F_{24}\boldsymbol{e_{24}}$$

Where; ($\boldsymbol{e_{11}}$, $\boldsymbol{e_{12}}$, $\boldsymbol{e_{13}}$, $\boldsymbol{e_{14}}$) ($\boldsymbol{e_{21}}$, $\boldsymbol{e_{22}}$, $\boldsymbol{e_{23}}$, $\boldsymbol{e_{24}}$) are unit vectors

(F_{11} , F_{12} , F_{13} , F_{14}) (F_{21} , F_{22} , F_{23} , F_{24}) are scalar components of force

Each vector has a magnitude; $|\boldsymbol{F_n}| = F_{n5}$ where; n = 1,2

The components are related to magnitude; $F_{n1}^2 + F_{n2}^2 + F_{n3}^2 + F_{n4}^2 = F_{n5}^2$

Sub-components (F_{n6}, F_{n7}) are also related; $F_{n6}^2 = F_{n5}^2 - F_{n4}^2 = F_{n3}^2 + F_{n7}^2$

$$F_{n7}^2 = F_{n1}^2 + F_{n2}^2 = F_{n6}^2 - F_{n3}^2$$

Component Geometry;

Each vector ($\boldsymbol{F_n}$) has components and sub-components arranged as angular geometry;

$F_{n1} = F_{n7}\text{Cos}(A_{n1})$ and; $F_{n2} = F_{n7}\text{Sin}(A_{n1})$

$F_{n7} = F_{n6}\text{Cos}(A_{n2})$ and; $F_{n3} = F_{n6}\text{Sin}(A_{n2})$

$F_{n6} = F_{n5}\text{Cos}(A_{n3})$ and; $F_{n4} = F_{n5}\text{Sin}(A_{n3})$

Radiant Conditions;

Three conditions are required for emission; Condition 1; $A_{11} = A_{12}$

Condition 2; $F_{11} = F_{26}$

Condition 3; $A_{13} = A_{23}$

The Force Equation;

From condition 1; $A_{11} = A_{12}$

$Cos(A_{11}) = Cos(A_{12})$

$F_{11}/F_{17} = F_{17}/F_{16}$

$F_{11}F_{16} = F_{17}^2$

From condition 2; $F_{26}F_{16} = F_{17}^2$

$F_{25}Cos(A_{23})F_{15}Cos(A_{13}) = F_{17}^2$

From condition 3; $F_{25}Cos(A_{13})F_{15}Cos(A_{13}) = F_{17}^2$

The force equation is; $F_{25}F_{15}Cos^2(A_{13}) = F_{17}^2$

Assume; $Sin(A_{13}) = ¼$ and; $Cos(A_{13}) = ¼(15)^{½}$

The scalar force equation may be written as;

$15F_{25}F_{15} = 16F_{17}^2$

Definitions of Force;

The scalar force equation relates the radiant force (F_{25}) to the dynamic force of the emitter (F_{15}) and the thermal force of the emitter (F_{17}). Components of force may be defined as;

$F_{15} = E_{15}/(2\pi r_{15})$ and; $F_{17} = E_{17}^2/hc$

Where; E_{15} is the dynamic energy of the emitter; $E_{15} = ½\hbar f_{15}$

f_{15} is the frequency of the emitter

ђ is the reduced Plank constant

r_{15} is the radius of the emitter; $r_{15} = v_{15}t$

v_{15} is the average vibrational velocity of the emitter and t is time

E_{17} is thermal energy; $E_{17} = ½\pi k_B T_{17}$

k_B is the Boltzmann constant

T_{17} is temperature

h is the Plank constant

c is the light constant

The Radiance Equation;

The scalar force equation is; $15F_{\mathbf{25}}F_{\mathbf{15}} = 16F_{\mathbf{17}}^2$

$15F_{\mathbf{25}}(1)F_{\mathbf{15}} = 16F_{\mathbf{17}}^2$

Definitions give; $15F_{\mathbf{25}}(v_{15}t/r_{15})F_{\mathbf{15}} = 16F_{\mathbf{17}}^2$

$15F_{\mathbf{25}}(v_{15}t/r_{15})(E_{15}/2\pi r_{15}) = 16E_{17}^4/h^2c^2$

$15F_{\mathbf{25}}(v_{15}t/r_{15})(\tfrac{1}{2}\hbar f_{15}/2\pi r_{15}) = 16(\tfrac{1}{2}\pi k_B T_{17})^4/h^2c^2$

$15F_{\mathbf{25}}(v_{15}t/r_{15})(\hbar f_{15}/4\pi r_{15}) = \pi^4 k_B^4 T_{17}^4/h^2c^2$

$15(F_{25}v_{15})\hbar(tf_{15})(1/4\pi r_{15}^2) = \pi^4 k_B^4 T_{17}^4/h^2c^2$

$15(F_{25}v_{15})(h/2\pi)(1)(1/4\pi r_{15}^2) = \pi^4 k_B^4 T_{17}^4/h^2c^2$

$15(F_{25}v_{15})(1/4\pi r_{15}^2) = 2\pi^5 k_B^4 T_{17}^4/h^3c^2$

Emissive power (P) is; $P = F_{25}v_{15}$

Emissive surface area (A_{15}) is; $A_{15} = 4\pi r_{15}^2$

Giving; $15P/A_{15} = (2\pi^5 k_B^4/h^3c^2)T_{17}^4$

Brightness (β) is; $\beta = P/A_{15}$

Giving the radiance equation; $\beta = (2\pi^5 k_B^4/15h^3c^2)T_{17}^4 = \sigma T_{17}^4$

Where; σ is the Stefan-Boltzmann constant; $\sigma = 2\pi^5 k_B^4/15c^2h^3$

Conclusion;

Two vectors represent forces associated with an emitter and radiation. If three “radiant conditions” apply, then the components of force will give the Stephan-Boltzmann equation of brightness.