41 Special Reciprocation

Two forces are reciprocal. Reciprocation requires three conditions:

- the forces act upon a common particle
- have equal strength
- act in opposite directions

Force may be represented as a vector. A 3D force vector may represent "instantaneous force".

Reciprocal forces may be represented as "reciprocal vectors".

The Force Vector;

Each force is represented as a 3D vector ($\boldsymbol{F_n}$). The origin (tail) of a vector corresponds to the center of a particle. The vectors are:

$$\boldsymbol{F_n} = F_{nx}\boldsymbol{i_n} + F_{ny}\boldsymbol{j_n} + F_{nz}\boldsymbol{k_n}$$

Where: n is a vector identifier (n = 1,2)

F_{nx}, F_{ny}, F_{nz} are components of force (Cartesian)

$\boldsymbol{i_n}$, $\boldsymbol{j_n}$, $\boldsymbol{k_n}$ are directional vectors (unit vectors)

Each vector has a unique frame of reference: the set ($\boldsymbol{i_n}$, $\boldsymbol{j_n}$, $\boldsymbol{k_n}$)

The vectors have magnitude: $|\boldsymbol{F}_n| = F_{nr}$

The magnitudes are related to components: $F_{nx}^2 + F_{ny}^2 + F_{nz}^2 = F_{nr}^2$

Sub-components (F_{nw}) are: $F_{nw}^2 = F_{nr}^2 - F_{nz}^2 = F_{nx}^2 + F_{ny}^2$

Vector Geometry;

The Cartesian co-ordinates of force are: F_{nx}, F_{ny}, F_{nz}

The Polar co-ordinates of force are: F_{nr}, θ_n, ϕ_n

Compliment angle (θ'_n) is: $\theta'_n = \frac{1}{2}\pi - \theta_n$

Vector geometry is: $F_{nx} = F_{nw}Cos(\phi_n)$ and: $F_{ny} = F_{nw}Sin(\phi_n)$

$F_{nw} = F_{nr}Cos(\theta'_n)$ and: $F_{nz} = F_{nr}Sin(\theta'_n)$

Reciprocation;

Reciprocal forces act upon the same particle. Two force vectors ($\boldsymbol{F_1}$, $\boldsymbol{F_2}$) are reciprocal.

The fundamental rules for reciprocal vectors are:

Rule 1: $|\boldsymbol{F_1}| = |\boldsymbol{F_2}|$ giving: $F_{1r} = F_{2r}$

Rule 2: $\phi_1 = \phi_2$

Rule 3: $\theta'_1 = \theta'_2$

Rule 4: $F_{1x} + F_{2x} = 0$ (reciprocal components)

Reciprocal geometry gives:

$F_{1r} - F_{2r} = 0$ and: $\phi_1 - \phi_2 = 0$ and: $\theta'_1 - \theta'_2 = 0$

$F_{1x} + F_{2x} = 0$ and: $F_{1y} + F_{2y} = 0$ and: $F_{1z} + F_{2z} = 0$

Special Reciprocation;

Special reciprocation requires one additional rule:

Rule 5: $\phi_1 = \theta'_1$ (giving: $\phi_1 = \phi_2 = \theta'_1 = \theta'_2$)

$\mathrm{Cos}(\phi_1) = \mathrm{Cos}(\theta'_1)$

From vector geometry: $F_{1x}/F_{1w} = F_{1w}/F_{1r}$

$F_{1x}F_{1r} = F_{1w}^2$ (also: $F_{2x}F_{2r} = F_{2w}^2$)

From rule 4: $F_{1x} + F_{2x} = 0$

$F_{1w}^2/F_{1r} + F_{2w}^2/F_{2r} = 0$

Rule 1 gives: $F_{1w}^2 + F_{2w}^2 = 0$

$F_{1w} = iF_{2w}$ where: $i^2 + 1 = 0$

Conclusion;

The complex constant (i) is considered to represent a ratio of special reciprocity: $i = F_{1w}/F_{2w}$