08 The Compton Distribution Ratio

The Compton Effect discovered in 1923 represents the interaction between a photon (x ray) and an electron. The interaction is elastic; the photon is "scattered", and the electron "recoils". The Compton equation relates a ratio of momentum to a scattering angle. The Compton equation also includes a "probability ratio", or "distribution ratio" which is related to the Lorentz distribution function.

The Compton Equation;

The Compton equation defines the change of wavelength of an x-ray photon after collision with an electron;

$$\lambda_2 - \lambda_1 = (h/m_e c)(1 - \text{Cos}\theta)$$

Where; λ_2 is the wavelength of the x-ray photon after the interaction

λ_1 is the wavelength of the x-ray photon before the interaction ($\lambda_2 > \lambda_1$)

h is the Plank constant

c is the light constant

m_e is the rest mass of an electron

θ is the scattering angle of the photon

It is convenient to represent the change in photonic wavelength (λ_3): $\lambda_3 = \lambda_2 - \lambda_1$

The Compton equation may also be written as a ratio of momentum: $p_e/p_3 = 1 - \text{Cos}(\theta)$

Where: $p_e = m_e c$

$p_3 = h/\lambda_3$ representing the momentum of an "exchange particle"

The Lorentz Distribution Function;

A "substance" is distributed over an infinite range of "distance". The distribution may be represented as a distribution wave (or bell-shaped curve). The curve is defined by a Lorentz distribution function (f_{L3}), which is a function of distance (x) from some point of reference. The function has three parameters (x_0,γ,I) and is defined as:

$$f_{L3} = I\gamma^2[(x - x_0)^2 + \gamma^2]^{-1}$$

Where: x is the "distribution distance" (from some reference point)

f_{L3} is the probable value of the "distributed substance" at the distribution distance

I is the "amplitude parameter" of the wave. It is the "probable peak value" of the distributed substance

x_0 is the "location parameter" which specifies the "x location" of the peak value of the distributed substance

γ is the "shape parameter" which specifies the shape of the distribution curve. A bell-curve has "sloping shape" that may range from "gentle" to "severe". The shape parameter (γ) is the "x location" of the half-maximum (½I).

It is appropriate to write: $\gamma = x_\gamma$

Giving: $f_{L3} = I x_\gamma^2[(x - x_0)^2 + x_\gamma^2]^{-1}$

It is also appropriate to write: $f_{L3} = y_x$ and: $I = y_{max}$

Giving: $y_x = y_{max} x_\gamma^2[(x - x_0)^2 + x_\gamma^2]^{-1}$

It is convenient to assume: $x_0 = 0$

The "standard Lorentz distribution" is: $y_x = y_{max} x_\gamma^2/(x^2 + x_\gamma^2)$

The Lorentz Probability Ratio;

A "standard probability ratio" (R_{L30}) is: $R_{L30} = y_x/y_{max} = x_\gamma^2/(x^2 + x_\gamma^2) = x_\gamma^2/x_R^2$

Where: $x_R^2 = x^2 + x_\gamma^2$

The standard probability ratio gives the probable ratio of distribution of a substance at some distance (x) from a point of reference, compared to unity.

If: $x = 0$, then: $R_{L30} = 1$

The Bell Curve;

The "distribution wave" or "bell-curve" may be represented by a variable point: (x, y_x)

The curve is an asymptote to: $y_x = 0$

Key points on the curve are: $(-\infty, 0)$, $(-x_\gamma, \frac{1}{2}y_{max})$, $(0, y_{max})$, $(x_\gamma, \frac{1}{2}y_{max})$, $(\infty, 0)$

The Compton Probability Ratio;

The "Compton Probability Ratio" (R_θ) is: $R_\theta = y_{\theta x}/y_{\theta max} = 2R_{L30} = 2x_\gamma^2/x_R^2$

$$R_\theta = y_{\theta x}/y_{\theta max} = 2x_\gamma^2/(x^2 + x_\gamma^2)$$

$$R_\theta = p_e/p_3 = 2x_\gamma^2/(x^2 + x_\gamma^2)$$

Where: $y_{\theta x}/y_{\theta max} = p_e/p_3$ (a ratio of momentum)(also may be energy, or force, or acceleration)

$p_e = m_e c$

$p_3 = h/\lambda_3$ representing the momentum of an "exchange particle"

Assume: $x_R{}^2 = x^2 + x_\gamma{}^2$ and: $x_\gamma = x\mathrm{Tan}(½\theta)$

Then: $2xx_\gamma = x_R{}^2\mathrm{Sin}(\theta)$ and: $x^2 - x_\gamma{}^2 = x_R{}^2\mathrm{Cos}(\theta)$

Giving: $1 - \mathrm{Cos}(\theta) = 2x_\gamma{}^2/(x^2 + x_\gamma{}^2) = 2x_\gamma{}^2/x_R{}^2$

The Compton equation may be written as: $R_\theta = y_{\theta x}/y_{\theta max} = 2R_{L30} = 2x_\gamma{}^2/x_R{}^2$

$R_\theta = p_e/p_3 = 1 - \mathrm{Cos}(\theta)$

Conclusion;

The Compton equation includes a "probability ratio" (distribution ratio) which is related to the "Lorentz probability ratio".