45 Rules of Direction

Assume two vectors may be multiplied in a variety of ways (dot product, cross product, and other products). Each vector contains a set of "basis vectors" (unit vectors). The basis vectors may also be identified as "directional vectors".

Each type of multiplier is represented as a "labelled general multiplier" (x_N).

Where: N is a "product type number" (product label).

N = 0 gives the dot product

N = 1 gives the cross product

N = 2 gives a "star product"

N > 2 gives other vector products

A set of four "directional rules" are associated with each type. The "rules of direction" are a convenient way to represent various types of vector multiplication.

The Vectors:

Two 3D vectors (***A***, ***B***) are:

$\boldsymbol{A} = A_1\boldsymbol{e_1} + A_2\boldsymbol{e_2} + A_3\boldsymbol{e_3}$

$\boldsymbol{B} = B_1\boldsymbol{e_1} + B_2\boldsymbol{e_2} + B_3\boldsymbol{e_3}$

Where: (A_1 , A_2 , A_3)(B_1 , B_2 , B_3) are scalar components of each vector

($\boldsymbol{e_1}$, $\boldsymbol{e_2}$, $\boldsymbol{e_3}$) are directional vectors (unit vectors)

The General Vector Product:

A "labelled general multiplier" (x_N) represents all types of product.

The general 3D product is: $\boldsymbol{A}\ x_N\ \boldsymbol{B} = \Sigma\ \text{Table}(x_N)$

Where: Table (x_N) =

$A_1B_1\boldsymbol{e_1}x_N\boldsymbol{e_1}$	$A_1B_2\boldsymbol{e_1}x_N\boldsymbol{e_2}$	$A_1B_3\boldsymbol{e_1}x_N\boldsymbol{e_3}$
$A_2B_1\boldsymbol{e_2}x_N\boldsymbol{e_1}$	$A_2B_2\boldsymbol{e_2}x_N\boldsymbol{e_2}$	$A_2B_3\boldsymbol{e_2}x_N\boldsymbol{e_3}$
$A_3B_1\boldsymbol{e_3}x_N\boldsymbol{e_1}$	$A_3B_2\boldsymbol{e_3}x_N\boldsymbol{e_2}$	$A_3B_3\boldsymbol{e_3}x_N\boldsymbol{e_3}$

Each type of multiplier (x_N) has four rules of direction.

The Dot Product:

The dot product of two vectors returns a scalar, it is represented if: N = 0

The “dot multiplier” may be represented as: x_0

The 3D dot product is: $\boldsymbol{A}\ x_0\ \boldsymbol{B} = \Sigma\ \text{Table}(x_0)$

Where: Table (x_0) =

$A_1B_1\boldsymbol{e_1}x_0\boldsymbol{e_1}$	$A_1B_2\boldsymbol{e_1}x_0\boldsymbol{e_2}$	$A_1B_3\boldsymbol{e_1}x_0\boldsymbol{e_3}$
$A_2B_1\boldsymbol{e_2}x_0\boldsymbol{e_1}$	$A_2B_2\boldsymbol{e_2}x_0\boldsymbol{e_2}$	$A_2B_3\boldsymbol{e_2}x_0\boldsymbol{e_3}$
$A_3B_1\boldsymbol{e_3}x_0\boldsymbol{e_1}$	$A_3B_2\boldsymbol{e_3}x_0\boldsymbol{e_2}$	$A_3B_3\boldsymbol{e_3}x_0\boldsymbol{e_3}$

The direction vectors are represented as: $(\boldsymbol{e_n}, \boldsymbol{e_p})$

The four rules of direction for Dot products are:

Rule 1: $\boldsymbol{e_n}x_0\boldsymbol{e_n} = 1$ Rule 2: $\boldsymbol{e_p}x_0\boldsymbol{e_p} = 1$ Rule 3: $\boldsymbol{e_n}x_0\boldsymbol{e_p} = 0$ Rule 4: $\boldsymbol{e_p}x_0\boldsymbol{e_n} = 0$

Applying the rules of direction gives: Table $(x_0)_R$ =

A_1B_1	0	0
0	A_2B_2	0
0	0	A_3B_3

Giving a dot product: $\boldsymbol{A}\ x_0\ \boldsymbol{B} = \boldsymbol{A.B} = A_1B_1 + A_2B_2 + A_3B_3$

The Cross Product:

The cross product returns a vector which is perpendicular to the plane of ***A*** and ***B***.

It is represented if: N = 1

The “cross multiplier” may be represented as: x_1

The 3D cross product is: $\boldsymbol{A}\ x_1\ \boldsymbol{B} = \Sigma\ \text{Table}(x_1)$

Where: Table (x_1) =

$A_1B_1\boldsymbol{e_1}x_1\boldsymbol{e_1}$	$A_1B_2\boldsymbol{e_1}x_1\boldsymbol{e_2}$	$A_1B_3\boldsymbol{e_1}x_1\boldsymbol{e_3}$
$A_2B_1\boldsymbol{e_2}x_1\boldsymbol{e_1}$	$A_2B_2\boldsymbol{e_2}x_1\boldsymbol{e_2}$	$A_2B_3\boldsymbol{e_2}x_1\boldsymbol{e_3}$
$A_3B_1\boldsymbol{e_3}x_1\boldsymbol{e_1}$	$A_3B_2\boldsymbol{e_3}x_1\boldsymbol{e_2}$	$A_3B_3\boldsymbol{e_3}x_1\boldsymbol{e_3}$

Rules of direction for cross products:

Rule 1: $\boldsymbol{e_n} _x \boldsymbol{e_n} = 0$ Rule 2: $\boldsymbol{e_p} _x \boldsymbol{e_p} = 0$ Rule 3: $\boldsymbol{e_n} _x \boldsymbol{e_p} = -\boldsymbol{e_0}^{n+p}\boldsymbol{e_{6-n-p}}$ Rule 4: $\boldsymbol{e_p} _x \boldsymbol{e_n} = -\boldsymbol{e_n} _x \boldsymbol{e_p}$

Where: $\boldsymbol{e_0} = -1$

Applying direction rules gives: Table $(x_1)_R$ =

0	$A_1B_2\boldsymbol{e_3}$	$-A_1B_3\boldsymbol{e_2}$
$-A_2B_1\boldsymbol{e_3}$	0	$A_2B_3\boldsymbol{e_1}$
$A_3B_1\boldsymbol{e_2}$	$-A_3B_2\boldsymbol{e_1}$	0

Giving: $\boldsymbol{AxB} = (A_2B_3 - A_3B_2)\boldsymbol{e_1} + (A_3B_1 - A_1B_3)\boldsymbol{e_2} + (A_1B_2 - A_2B_1)\boldsymbol{e_3}$

The Star Product:

The star product returns a vector which is not perpendicular to the plane of ***A*** and ***B***.

It is represented if: N = 2

The "star multiplier" may be represented as: x_2

The 3D star product is: $\boldsymbol{A}\ x_2\ \boldsymbol{B} = \boldsymbol{A}*\boldsymbol{B} = \Sigma$ Table(x_2)

Where: Table (x_2) =

$A_1B_1\boldsymbol{e_1}x_2\boldsymbol{e_1}$	$A_1B_2\boldsymbol{e_1}x_2\boldsymbol{e_2}$	$A_1B_3\boldsymbol{e_1}x_2\boldsymbol{e_3}$
$A_2B_1\boldsymbol{e_2}x_2\boldsymbol{e_1}$	$A_2B_2\boldsymbol{e_2}x_2\boldsymbol{e_2}$	$A_2B_3\boldsymbol{e_2}x_2\boldsymbol{e_3}$
$A_3B_1\boldsymbol{e_3}x_2\boldsymbol{e_1}$	$A_3B_2\boldsymbol{e_3}x_2\boldsymbol{e_2}$	$A_3B_3\boldsymbol{e_3}x_2\boldsymbol{e_3}$

Rules of direction for star products:

Rule 1: $\boldsymbol{e_n}x_2\boldsymbol{e_n} = \boldsymbol{e_{4-n}}$ Rule 2: $\boldsymbol{e_p}x_2\boldsymbol{e_p} = -\boldsymbol{e_{4-p}}$ Rule 3: $\boldsymbol{e_n}x_2\boldsymbol{e_p} = 0$ Rule 4: $\boldsymbol{e_p}x_2\boldsymbol{e_n} = 0$

Applying direction rules gives:

$A_1B_1\boldsymbol{e_3}$	0	0
0	$A_2B_2\boldsymbol{e_2}$	0
0	0	$A_3B_3\boldsymbol{e_1}$

Giving: $\boldsymbol{A}*\boldsymbol{B} = A_3B_3\boldsymbol{e_1} + A_2B_2\boldsymbol{e_2} + A_1B_1\boldsymbol{e_3}$

Conclusion:

Vector products of various types are defined by rules of direction. A variety of products may be of use in physics. The star product may represent a gravitational field.