40 Schwarzschild Vectors

The Schwarzschild metric may be simply obtained from the dot product of two vectors of acceleration ("Schwarzschild vectors"), no Christoffel symbols, no tensors. Each vector is four dimensional.

The dot product represents an "interaction of acceleration", the result is a scalar field equation. Groupings and definitions of velocities, radial distances, and operators will give the Schwarzschild metric.

The Schwarzschild Metric;

The Schwarzschild metric may be written as:

$$\partial s^2 = -c^2\partial T^2 = -(R/r)c^2\partial t^2 + (r/R)\partial r^2 + r^2\partial\theta^2 + r^2\mathrm{Sin}^2(\theta)\partial\phi^2$$

Where: s is the space-time interval

T is "proper time" (may also be associated with frequency)

t is "co-ordinate time"

θ, ϕ are spatial angles

r is radial distance

R is radial difference; $R = r - r_S$

r_S is the Schwarzschild radius; $r_S = 2Gm/c^2$

c is the light constant

G is the gravitational constant

m is mass

Spatial Dimensions;

The dimension set (x, y, z) represents the Cartesian co-ordinates of space.

The dimension set (r, θ, ϕ) represents the Polar co-ordinates of space.

The angles (θ, ϕ) are: $\theta = U_\theta/r$ and: $\phi = U_\phi/w$

Where: U_θ, U_ϕ represent arc lengths

w is a radial distance; $w^2 = x^2 + y^2$

r is a radial distance; $r^2 = x^2 + y^2 + z^2$

The dimension sets are related.

$x = w\text{Cos}(\phi)$ and: $y = w\text{Sin}(\phi)$ and: $z = r\text{Cos}(\theta)$ and: $w = r\text{Sin}(\theta)$

$U_\theta = r\theta$ and: $\partial U_\theta = r\partial\theta$ and: $U_\phi = w\phi$ and: $\partial U_\phi = w\partial\phi = r\text{Sin}(\theta)\partial\phi$

The Schwarzschild metric may also be written as: $(R/r)c^2\partial t^2 = (r/R)\partial r^2 + \partial U_\theta{}^2 + \partial U_\phi{}^2 + c^2\partial T^2$

Acceleration Vectors;

Acceleration may be represented as a vector. Two types of acceleration are required. Each type is represented as a vector ($\boldsymbol{g_1}$, $\boldsymbol{g_2}$). Each vector is four dimensional.

An acceleration vector ($\boldsymbol{g_n}$) is: $\boldsymbol{g_n} = g_{n1}\boldsymbol{e_1} + g_{n2}\boldsymbol{e_2} + g_{n3}\boldsymbol{e_3} + g_{n4}\boldsymbol{e_4}$

Where: g_{nN} is a component of acceleration

n is a vector identifier (n = 1,2)

N is a term identifier (N = 1,2,3,4)

$\boldsymbol{e_N}$ is a basis vector (unit vector)

The vector has magnitude: $|\boldsymbol{g_n}| = g_{n5}$

The magnitude is related to components: $g_{n1}{}^2 + g_{n2}{}^2 + g_{n3}{}^2 + g_{n4}{}^2 = g_{n5}{}^2$

Interaction;

Two types of acceleration interact in space-time. Each type of acceleration is represented as a four-dimensional vector ($\boldsymbol{g_1}$, $\boldsymbol{g_2}$) which act upon a common point in space-time and are separated by an "angle of interaction" (β). The interaction is represented as a dot product of the vectors ($\boldsymbol{g_1}\cdot\boldsymbol{g_2}$):

$$\boldsymbol{g_1}\cdot\boldsymbol{g_2} = g_{15}g_{25}\text{Cos}(\beta) = g_{11}g_{21} + g_{12}g_{22} + g_{13}g_{23} + g_{14}g_{24}$$

A component of acceleration (g_{nN}) may be defined as: $g_{nN} = v_{nN}{}^2/x_{nN}$

Where: v_{nN} is velocity and: x_{nN} is displacement

Giving a scalar "equation of interaction":

$$(v_{15}{}^2/x_{15})(v_{25}{}^2/x_{25})\text{Cos}(\beta) = (v_{11}{}^2/x_{11})(v_{21}{}^2/x_{21}) + (v_{12}{}^2/x_{12})(v_{22}{}^2/x_{22}) + (v_{13}{}^2/x_{13})(v_{23}{}^2/x_{23}) + (v_{14}{}^2/x_{14})(v_{24}{}^2/x_{24})$$

The Scalar Field Equation;

Assume: $R_{2N} = x_{2N}\text{Cos}(\beta)$

Giving a "scalar field equation":

$$(v_{15}{}^2/x_{15})(v_{25}{}^2/x_{25}) = (v_{11}{}^2/x_{11})(v_{21}{}^2/R_{21}) + (v_{12}{}^2/x_{12})(v_{22}{}^2/R_{22}) + (v_{13}{}^2/x_{13})(v_{23}{}^2/R_{23}) + (v_{14}{}^2/x_{14})(v_{24}{}^2/R_{24})$$

Groupings of Radial Distance;

Two types of radial distance (r,R) are required. Associated radial distances (x_{nN} ,R_{nN}) may be grouped by type:

$R = x_{11}, R_{21}, R_{22}, R_{23}, R_{24}$

$r = x_{15}, x_{25}, x_{12}, x_{13}, x_{14}$

The scalar field equation may be written as:

$$(v_{15}^2/r)(v_{25}^2/r) = (v_{11}^2/R)(v_{21}^2/R) + (v_{12}^2/r)(v_{22}^2/R) + (v_{13}^2/r)(v_{23}^2/R) + (v_{14}^2/r)(v_{24}^2/R)$$

Re-arranging radials gives:

$$(R/r)v_{15}^2v_{25}^2 = (r/R)v_{11}^2v_{21}^2 + v_{12}^2v_{22}^2 + v_{13}^2v_{23}^2 + v_{14}^2v_{24}^2$$

Velocity Definitions;

Two types of velocity (v_{1N} , v_{2N}) are required.

"Pathwise velocity" (v_{1N}) is the product of "path displacement" (u_{1N}) and a time-cycle operator ($\partial/\partial t$):

$$v_{1N} = \partial u_{1N}/\partial t$$

Path displacement may be linear or curved.

"Wave velocity" (v_{2N}) is the product of "wavelength" (λ_{2N}) and a frequency operator (f):

$$v_{2N} = f\lambda_{2N} = \partial\lambda_{2N}/\partial T$$

The scalar field equation may include velocity definitions:

$$(R/r)(\partial u_{15}^2/\partial t^2)(\partial\lambda_{25}^2/\partial T^2) = (r/R)(\partial u_{11}^2/\partial t^2)(\partial\lambda_{21}^2/\partial T^2) + (\partial u_{12}^2/\partial t^2)(\partial\lambda_{22}^2/\partial T^2) + (\partial u_{13}^2/\partial t^2)(\partial\lambda_{23}^2/\partial T^2) + (\partial u_{14}^2/\partial t^2)(\partial\lambda_{24}^2/\partial T^2)$$

Products of velocity may swap components: $(\partial u_{14}^2/\partial t^2)(\partial\lambda_{24}^2/\partial T^2) = (\partial u_{14}^2/\partial T^2)(\partial\lambda_{24}^2/\partial t^2)$

Giving: $(R/r)(\partial u_{15}^2/\partial t^2)(\partial\lambda_{25}^2/\partial T^2) = (r/R)(\partial u_{11}^2/\partial t^2)(\partial\lambda_{21}^2/\partial T^2) + (\partial u_{12}^2/\partial t^2)(\partial\lambda_{22}^2/\partial T^2) +$

$(\partial u_{13}^2/\partial t^2)(\partial\lambda_{23}^2/\partial T^2) + (\partial u_{14}^2/\partial T^2)(\partial\lambda_{24}^2/\partial t^2)$

Special velocities are: $\partial u_{15}/\partial t = \partial u_{14}/\partial T = c$

A "general scalar field equation" may be written as:

$$(R/r)c^2(\partial\lambda_{25}^2/\partial T^2) = (r/R)(\partial u_{11}^2/\partial t^2)(\partial\lambda_{21}^2/\partial T^2) + (\partial u_{12}^2/\partial t^2)(\partial\lambda_{22}^2/\partial T^2) + (\partial u_{13}^2/\partial t^2)(\partial\lambda_{23}^2/\partial T^2) + c^2(\partial\lambda_{24}^2/\partial t^2)$$

Grouping of Wavelength;

Assume all wavelengths belong to a single group: $\lambda_{nN} = \lambda$

Giving a "simplified field equation":

$$(R/r)c^2(\partial\lambda^2/\partial T^2) = (r/R)(\partial u_{11}{}^2/\partial t^2)(\partial\lambda^2/\partial T^2) + (\partial u_{12}{}^2/\partial t^2)(\partial\lambda^2/\partial T^2) +$$

$$(\partial u_{13}{}^2/\partial t^2)(\partial\lambda^2/\partial T^2) + c^2(\partial\lambda^2/\partial t^2)$$

Removing incremental wavelength gives:

$$(R/r)c^2(1/\partial T^2) = (r/R)(\partial u_{11}{}^2/\partial t^2)(1/\partial T^2) + (\partial u_{12}{}^2/\partial t^2)(1/\partial T^2) + (\partial u_{13}{}^2/\partial t^2)(1/\partial T^2) + c^2(1/\partial t^2)$$

The Reduced Metric;

Re-arranging time components gives a "reduced metric":

$$(R/r)c^2\partial t^2 = (r/R)\partial u_{11}{}^2 + \partial u_{12}{}^2 + \partial u_{13}{}^2 + c^2\partial T^2$$

The Schwarzschild Metric;

Incremental path-wise displacement may be defined as:

$\partial u_{11} = \partial r$ and: $\partial u_{12} = \partial U_\theta$ and: $\partial u_{13} = \partial U_\phi$

Giving the Schwarzschild metric:

$$(R/r)c^2\partial t^2 = (r/R)\partial r^2 + \partial U_\theta{}^2 + \partial U_\phi{}^2 + c^2\partial T^2$$

$$(R/r)c^2\partial t^2 = (r/R)\partial r^2 + r^2\partial\theta^2 + r^2\mathrm{Sin}^2(\theta)\partial\phi^2 + c^2\partial T^2$$

Conclusion;

A dot product of two vectors of acceleration represents an "interaction of acceleration", the result is a scalar field equation. Groupings and definitions of velocities, radial distances, and operators will give the Schwarzschild metric.