A trivial disproof of special relativity?

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Abstract

It is a basic requirement of special relativity theory (SRT) that all relatively moving inertial observers agree on an observable (interference fringe shift, for example). It is shown that SRT trivially leads to a disagreement on the observables (interference fringe shift) in two relatively moving inertial reference frames.

Introduction

It is a basic requirement of special relativity theory (SRT) that all relatively moving inertial observers agree on an observable (an interference fringe shift, for example) [1]. We show that SRT trivially leads to a disagreement on the observables (interference fringe shift) in two relatively moving inertial reference frames.

Fringe shift predicted in two relatively moving inertial reference frames

Consider two inertial reference frames S and S' , with origins O and O' respectively (see figure on next page). S' is moving with velocity v relative to S, in the + x direction. S is the reference frame of the laboratory. At time $t = 0$, the origins O and O' coincide. An observer A with an interferometer is moving with velocity v_0 in the lab frame S , in the +x direction and is just passing through the origin O at $t = 0$. The light source is moving with velocity v_0 to the left and just passing through point E at $t = 0$. (The experiment would be easier to understand if the light source is fixed at point E. We have assumed a moving source to avoid Doppler effect.)

The difference form of the Lorentz transformation equations is given below.

$$\Delta x' = \gamma \ (\Delta x - v \, \Delta t \ )$$

$$\Delta t' = \gamma \ (\Delta t - \frac{v \Delta x}{c^2} \ )$$

where

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Δx is the difference in path lengths of the two light beams in frame S, and $\Delta x'$ is the difference in the path lengths of the two light beams in frame S'. Δt is the difference in the time of arrival of the two light beams in frame S, and $\Delta t'$ is the difference in time of arrival of the two light beams in frame S'.

Suppose that the interference fringe shift of the light speed experiment predicted in frame S is N, and the fringe shift predicted in frame S' for the same experiment is N'. It is a requirement of relativity theory that there should be an agreement on the observable (the fringe shift) in both frames, i.e. $N = N'$. Let us see if this is actually the case.

We know that,

$$N = \frac{c\,\Delta t}{\lambda} \qquad and \quad N' = \frac{c\,\Delta t'}{\lambda'}$$

But, due to time dilation [1]

$$\lambda' = \gamma\,\lambda$$

Therefore,

$$N' = \frac{c\,\Delta t'}{\lambda'} = \frac{c\,\gamma\,\left(\Delta t - \frac{v\Delta x}{c^2}\right)}{\gamma\lambda} = \frac{c\,\left(\Delta t - \frac{v\Delta x}{c^2}\right)}{\lambda} = \frac{c\;\Delta t}{\lambda} - \frac{\frac{v\Delta x}{c}}{\lambda} = N - \frac{v\Delta x}{c\lambda} \neq N$$

Galilean relativity, however, does not lead to such disagreement, as shown below.

$$\Delta x' = \Delta x - v\,\Delta t$$

$$\Delta t' = \Delta t$$

$$c' = c \pm V \;\Rightarrow\; \frac{c'}{c} = 1 \pm \frac{v}{c}$$

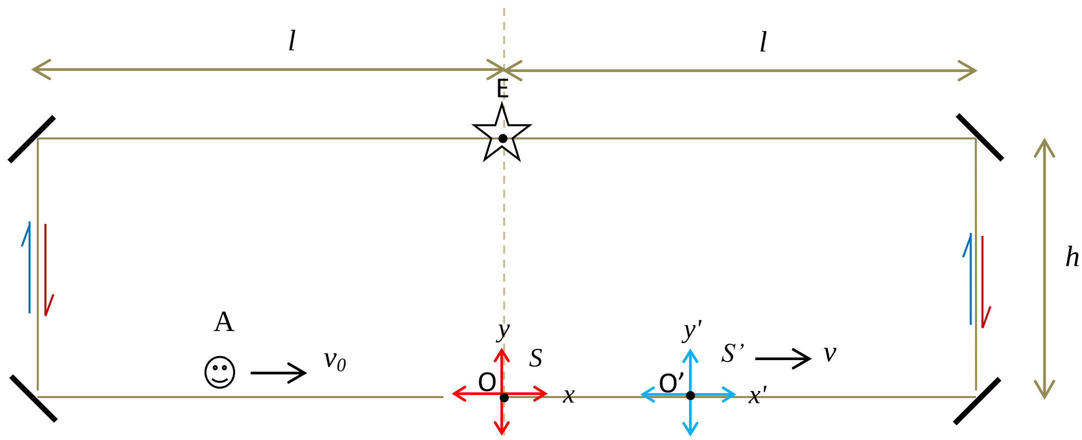
$$f' = f \quad\Rightarrow\; \frac{c'}{\lambda'} = \frac{c}{\lambda} \;\;\Rightarrow\; \lambda' = \lambda\,(1 \pm \frac{v}{c})$$

Therefore,

$$N' = \frac{c'\,\Delta t'}{\lambda'} = \frac{(c \pm v)\,\Delta t}{\lambda(1 \pm \frac{v}{c})} = \frac{c\,\Delta t}{\lambda} = N$$

Consider the following hypothetical experiment for illustration.

In the lab frame S, the moving observer A detects the clockwise propagating light at (x_2, t_2) and the counter-clockwise propagating light at (x_3, t_3).



In frame S, for the clockwise propagating light:

$$\frac{2l + h - x_2}{c} = \frac{x_2}{v_0}$$

$$\Rightarrow x_2 = \frac{v_0\,(2l + h)}{c + v_0}$$

and

$$t_2 = \frac{x_2}{v_0} = \frac{(2l + h)}{c + v_0}$$

For the counter-clockwise propagating light:

$$\frac{2l + h + x_3}{c} = \frac{x_3}{v_0}$$

$$\Rightarrow \quad x_3 = \frac{v_0\,(2l + h)}{c - v_0}$$

and

$$t_3 = \frac{x_3}{v_0} = \frac{(2l + h)}{c - v_0}$$

$$\Delta x = x_3 - x_2 = v_0\,(2l + h)\,\frac{2v_0}{c^2 - v^2}$$

$$\Delta t = t_3 - t_2 = (2l + h)\,\frac{2v_0}{c^2 - v^2}$$

The fringe shift as predicted in frame S can be written as:

$$N = \frac{\Delta x}{\lambda} = \frac{v_0\,(2l + h)\,\frac{2v_0}{c^2 - v^2}}{\lambda}$$

But

$$\Delta x' = \gamma\,(\Delta x - v\,\Delta t)$$

$$\Rightarrow \Delta x' = \gamma\,\left(v_0 - v\,(2l + h)\,\frac{2v_0}{c^2 - v^2}\right)$$

$$\Rightarrow \Delta x' = \gamma\,(2l + h)\frac{2v_0}{c^2 - v^2}\,(v_0 - v)$$

The fringe shift predicted in frame S’ will be:

$$N' = \frac{\Delta x'}{\lambda'} = \frac{\gamma\,(2l + h)\frac{2v_0}{c^2 - v^2}\,(v_0 - v)}{\gamma\,\lambda} = \frac{(2l + h)\frac{2v_0}{c^2 - v^2}\,(v_0 - v)}{\lambda}$$

We already obtained,

$$N = \frac{\Delta x}{\lambda} = \frac{v_0\ (2l + h)\ \frac{2v_0}{c^2 - v^2}}{\lambda}$$

We can see that,

$$N' \neq N$$

References

1. *Kennedy-Thorndike experiment*, Wikipedia