08 Compton Interaction

The interaction between an electron and a photon is represented by the Compton equation, published in 1923. Energy is transferred from the photon to the electron. The collision is elastic and each particle recoils in a different direction.

The interaction may be represented as three “phases” of momentum. Each phase (initial, exchange, and final) may be represented as a 3D vector of momentum.

The vectors must be “compatible” for energy transfer so that compatibility conditions apply to the vectors.

Vector components will give the Compton equation.

The Compton Equation;

The Compton equation is; $\lambda_2 - \lambda_1 = (1 - Cos_\theta)(h/m_e c)$

Where; θ is the scattering angle

λ_1 is the initial wavelength of the photon

λ_2 is the wavelength of photon after scattering ($\lambda_2 > \lambda_1$)

m_e is electron rest mass

h is the Plank constant

c is the light constant

Vectors of Momentum;

Three vectors of momentum ($\boldsymbol{p_1}$, $\boldsymbol{p_2}$, $\boldsymbol{p_3}$) are required. Generally a 3D vector of momentum ($\boldsymbol{p_n}$) is;

$$\boldsymbol{p_n} = p_{n1}\boldsymbol{e_{n1}} + p_{n2}\boldsymbol{e_{n2}} + p_{n3}\boldsymbol{e_{n3}}$$

Where; ‘n’ is a vector identifier (n = 1,2,3)

$\boldsymbol{p_1}$ is the intial phase

$\boldsymbol{p_2}$ is the exchange phase

$\boldsymbol{p_3}$ is the final phase

$\boldsymbol{e_{n1}}$, $\boldsymbol{e_{n2}}$, $\boldsymbol{e_{n3}}$ are basis vectors (orthogonal unit vectors)

p_{n1} , p_{n2} , p_{n3} are components of momentum

Each vector has a unique frame of reference.

Each vector has a magnitude; $|\boldsymbol{p_n}| = p_{n4}$

The scalar components are related to the magnitude; $p_{n1}^2 + p_{n2}^2 + p_{n3}^2 = p_{n4}^2$

A sub-component of momentum (p_{n5}) is; $p_{n5}^2 = p_{n1}^2 + p_{n2}^2 = p_{n4}^2 - p_{n3}^2$

Angular Geometry;

Each vector has "component angles" (A_{n1}, A_{n2}) having geometry;

$p_{n1} = p_{n5}Cos(A_{n1})$ and $p_{n2} = p_{n5}Sin(A_{n1})$

$p_{n5} = p_{n4}Cos(A_{n2})$ and $p_{n3} = p_{n4}Sin(A_{n2})$

$p_{15}^2 - p_{13}^2 = p_{14}^2Cos(2A_{12})$ and $2p_{15}p_{13} = p_{14}^2Sin(2A_{12})$

Compatibility;

All vectors must be compatible for interaction. The eight compatibility rules are;

$A_{11} = A_{12}$	$A_{21} = A_{22}$	$A_{31} = A_{32}$	$2A_{12} = \theta$
$p_{14} = p_{21}$	$p_{24} = p_{34}$	$p_{13} = p_{22}$	$P_{15} = p_{35}$

Where; θ is the scattering angle

Compatibility rules give an important result;

$A_{n1} = A_{n2}$

$Cos(A_{n1}) = Cos(A_{n2})$

$p_{n1}/p_{n5} = p_{n5}/p_{n4}$

$p_{n1}p_{n4} = p_{n5}^2$

Conservation of momentum may be represented as; $p_{15} = p_{35}$

$p_{15}^2 = p_{35}^2$

$p_{11}p_{14} = p_{31}p_{34}$

The Exchange Phase;

The exchange vector has input momentum (p_{21}) and output momentum (p_{24}).

Exchange performance may be represented as the "exchange ratio" which is "output/input"; p_{24}/p_{21}

Compatibility rules and geometry apply, giving;

$$p_{24}/p_{21} = p_{24}/p_{14} = p_{24}p_{11}/p_{14}p_{11} = p_{24}p_{11}/p_{15}^2 = p_{24}p_{11}/p_{35}^2 = p_{34}p_{11}/p_{35}^2$$

$$p_{24}/p_{21} = p_{34}p_{11}/p_{35}^2 = (p_{34}p_{11})/(p_{31}p_{34})$$

giving the equivalent exchange ratio (p_{11}/p_{31});

$$p_{24}/p_{21} = p_{11}/p_{31}$$

The Momentum Equation;

From angular geometry; $\mathrm{Cos}(\theta) = \mathrm{Cos}(2A_{12}) = (p_{15}^2 - p_{13}^2)/p_{14}^2$

$1 - \mathrm{Cos}(\theta) = (p_{14}^2 - p_{15}^2 + p_{13}^2)/p_{14}^2$

From compatibility rules; $1 - \mathrm{Cos}(\theta) = (p_{21}^2 - p_{15}^2 + p_{22}^2)/p_{14}^2$

From components; $1 - \mathrm{Cos}(\theta) = (p_{25}^2 - p_{15}^2)/p_{14}^2$

From angular rules; $1 - \mathrm{Cos}(\theta) = (p_{21}p_{24} - p_{11}p_{14})/p_{14}^2$

From compatibility rules; $1 - \mathrm{Cos}(\theta) = (p_{21}p_{24} - p_{11}p_{21})/p_{21}^2$

$1 - \mathrm{Cos}(\theta) = p_{24}/p_{21} - p_{11}/p_{21}$

The exchange ratio gives; $1 - \mathrm{Cos}(\theta) = p_{11}/p_{31} - p_{11}/p_{21}$

The momentum equation is; $1 - \mathrm{Cos}(\theta) = p_{11}(1/p_{31} - 1/p_{14})$

Momentum Definitions;

Momentum definitions are; $p_{11} = m_ec$

$p_{31} = h/\lambda_{31}$

$p_{14} = h/\lambda_{14}$

$p_{15} = m_xc = (p_{11}p_{14})^{½} = (hm_ec/\lambda_{14})^{½}$

$p_{12} = m_xv_x$

The momentum equation is; $1 - \mathrm{Cos}(\theta) = p_{11}(1/p_{31} - 1/p_{14})$

Definitions give; $1 - \mathrm{Cos}(\theta) = m_ec(\lambda_{31}/h - \lambda_{14}/h)$

Equivalent Compton equation; $1 - \mathrm{Cos}(\theta) = (m_ec/h)(\lambda_{31} - \lambda_{14})$

Mass dilation may also be represented;

$Cos(A_{11}) = p_{11}/p_{15} = m_e/m_x$ and $Sin(A_{11}) = p_{12}/p_{15} = v_x/c$

$Cos^2(A_{11}) + Sin^2(A_{11}) = 1$

$(m_e/m_x)^2 + (v_x/c)^2 = 1$

Conclusion;

The Compton Effect is a unique interaction which may be represented by three vectors of momentum. Each vector represents a "phase of interaction". Vector components give the Compton equation and mass dilation.